

AUTOMATED OPTIMUM DESIGN OF TALL MULTI-LEVEL GUYED TOWERS

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in Partial Fulfilment of the Requirements
for the Degree of
MASTER OF TECHNOLOGY

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By
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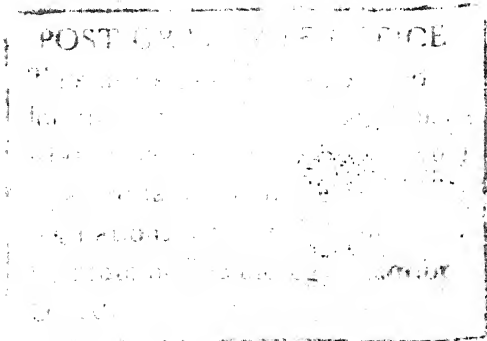
CERTIFICATE

Certified that the work presented in this thesis entitled 'AUTOMATED OPTIMUM DESIGN OF TALL MULTI-LEVEL GUYED TOWERS' by Mr. I.S. Sarma has been carried out under my supervision and it has not been submitted elsewhere for a degree.

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ABSTRACT

An automated optimum design of tall multi-level guyed towers is presented herein. Minimum cost design of multi-level guyed towers in a stress and deflection controlled space under elastic design philosophy has been carried out. The non-linear structural analysis of the system is performed by taking into account that the tower shaft is a beam-column resting on non-linear flexible supports. The geometry of the guy profile has been taken as a parabola. Large deformation effects have not been taken into account in the present work.

For the purposes of structural analysis the guys have been modelled as non-linear springs. The tower is considered as a beam-column hinged or fixed at the base and supported by non-linear springs at guy levels. The analysis is initiated by allowing the tower to undergo some fictitious initial displacements at the guy levels. Equilibrium equations are then written in the wind ward direction and in the direction normal to it at each guy level. Simultaneous linear equations, so obtained, are solved to obtain another set of fictitious but refined displacements. The process is iterated till the

deflections converge. In each iteration, the stiffness coefficients of the guys and the tower get modified. The deformed geometry corresponding to converged displacements represents the final deflected shape of the structural system considered herein.

Automated optimum design problem has been formulated considering the design variables: 1. the guy slopes, 2. the initial guy tensions, 3. the guy diameters, 4. the heights of the guy levels and 5. the areas of cross section of the l_g members and bracing members of the tower. Response of the system has been restricted by imposing constraints on the maximum displacements of the guyed tower. The maximum stress in the members of the tower and in the guy ropes is not allowed to increase a permissible limit. It is further ensured that the members of the tower are safe against buckling. Round bars for l_g members and equal angles for bracings have been chosen from amongst the available standard sections. The design variables are allowed to continuously vary within the range of specified lower and upper bounds. The objective function in the present work is to minimise the cost of the guyed

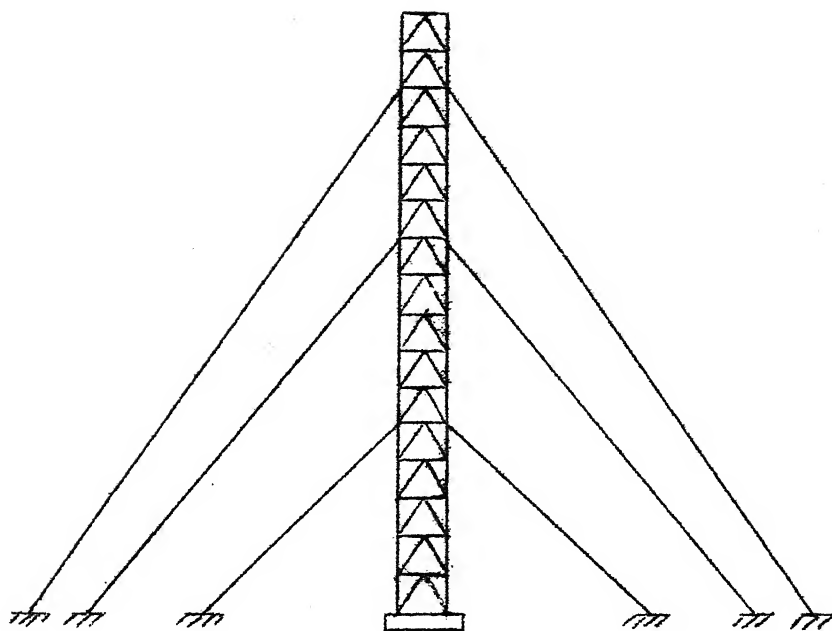
tower. Automated optimum design problem is formulated as a mathematical programming problem. An interior penalty function method using sequential unconstrained minimization technique is carried out by using Powell's algorithm and Golden Section Search is carried out for linear minimization. General computer programs for non-linear analysis and optimum design have been developed. Optimum design for a three level 100 m high triangular tower has been obtained from two different starting points. Existence of local minima is indicated by the numerical results.

CHAPTER I

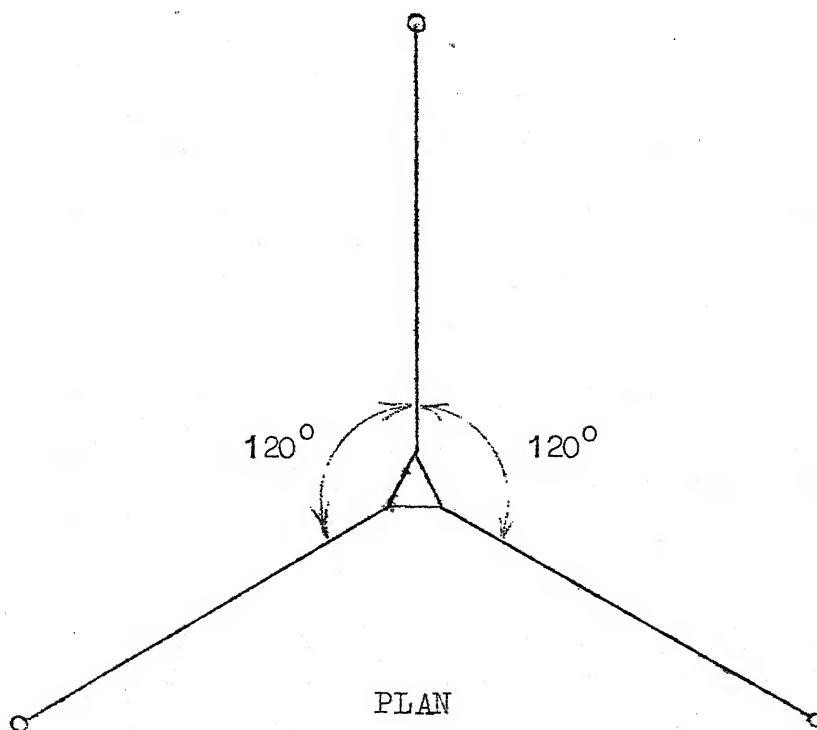
INTRODUCTION

The growth in the communication industry has resulted in the development of sophisticated know how for the design and erection of antenna supporting structures. With the stress on the relaying and broadcasting centers to cater for larger areas, the antenna supporting structures are getting taller. One has to choose between a guyed tower and a self supporting tower for these purposes. A self supporting tower is a three dimensional framed structure without any intermediate support. It is observed that self supporting towers are uneconomical for heights beyond 75 meters. The guyed towers are more economical for greater heights. However, they require large ground space.

Guyed towers are slender framed structures fixed or hinged at the base and supported at intermediate points by guys which are anchored to the ground. Guys are high tensile steel wires under some initial tension. A typical tall guyed tower is shown in Fig. 1.1. The number of levels of a guyed tower is the number of joints at which the guys are attached to the tower.



FRONT ELEVATION



PLAN

FIG. 1.1 GUYED TOWER (TRIANGULAR IN PLAN)

The response of a guyed tower is non-linear even under elastic conditions. This non-linearity is due both to the stiffening of the guy cables with increasing tension and to the destabilizing effect of the axial thrust in the tower itself. Since the axial thrust is partly due to the vertical components of the guy tensions, the two causes of non-linear behaviour are coupled together. Moreover, finite changes in over-all geometry may also produce non-linear behaviour.

Extreme care has to be exercised in taking the design decisions due to the non-linearities associated with the performance. There are too many design variables which have to be carefully fixed to arrive at a satisfactory design. Once a satisfactory design is at hand, considerations of generating an economical design require a systematic variation of design variables. Hence, the problem is most amenable to be formulated as an optimum design problem.

1.2 LITERATURE SURVEY

Multi-level guyed towers have been built since World War - I. As these were used mostly for military purposes in early times, not much of published material

is available on the design of such structures. However, there are some publications which deal with isolated aspects of the analysis and design of such structures. A review of these publications is made in this section.

Alton [2] has given a graphic solution for determining the spring constants of guys and has carried out an approximate analysis of the tower and guys under wind loading. Fiesenheiser [8] has described a scheme for the design of multi-level guyed towers. There are too many simplifications in the equations describing the behaviour of guys for the design. It is more of use to arrive at an initial design for a more rigorous analysis.

Cohen and Perrin [5] have written the general equations for horizontal guys as functions of sag normal to the chord of the inclined guys. These equations are used for inclined guys. The guys are converted into linear springs and the analysis of tower has been carried out as a beam-column resting on elastic supports. Analysis of the tower and the guys are approximate. Scott and Thurston [25] have considered the nonlinear effects of variation of the flexural stiffness of the tower due to axial loads under

symmetric loading.

Rowe [23] has formulated an approach where in the stresses are calculated first by analysing the tower as a beam-column and then amplifying them to account for the slack stresses in the guys. The charts for calculating the amplification factors are presented. Abegg [1] has presented interaction diagrams for stresses and displacements of the tower as functions of the cross sectional dimensions of the tower shaft. Hull [11] has developed a method for stability analysis of a guyed tower under symmetric loading conditions. Results have been expressed in terms of critical moment of inertia corresponding to a critical wind load.

Poskitt and Livesly [18] have described an iterative procedure for analysing the guyed towers, wherein, linearised slope deflection equations were solved in each iteration in the process of arriving at the final solution. Goldberg and Mayers [10] have derived non-linear force-deformation relations for the tower spans and guys. Algebraic equations are written for each joint of the tower. To simplify the

solution seeking, these equations have been delineated into a set of linear differential equations in the field of a wind load parameter (Square of the wind velocity). Odley [17] has developed an iterative procedure for analysing the guyed towers by replacing the guys as nonlinear springs based on the assumed displacements of the tower. Simultaneous linear algebraic equations are written for each joint and are solved to get a set of refined displacements. The guys are treated as parabolas and the effects of large deformation of the tower are neglected.

Livesly [15] has described a procedure for obtaining the optimum guy tensions to satisfy the deflection constraints. He minimized a linear function of guy erection tensions using the linear programming technique to arrive at the optimum values of the guy tensions. Reichelt, Brown and Melion [21] have described an on-line remote terminal computer system for use in the design of guyed towers. However, its use is limited to the choice of the computer system.

Chu Kuang and Chiun Ma [4] have analysed the guyed towers without considering the wind forces on the guys. Naqvi [16] has presented a method of

non-linear analysis of guyed towers using the stiffness method. The effect of geometric non-linearities arising due to the beam-column and large deformation effects on the non-linear analysis of the guyed towers has been studied. He concludes that, for a reasonably accurate analysis of the guyed tower, it would be sufficient to consider only the beam column effects and that the effect of large deformations of the tower on its over-all performance is only marginal.

1.3 SCOPE OF THE PRESENT WORK

As is clear from the literature survey, most of the efforts of the earlier investigators on the subject have gone towards developing the methods of analysis for the guyed towers. Little attention has been paid to the economical design of guyed towers. Practically no effort seems to have gone towards the study of the optimum design of guyed towers.

The minimum cost design of tall guyed towers may be formulated as an optimum design problem. The possible design variables could be 1. the number of guy levels, 2. the guy slope, 3. the initial guy tensions,

4. the guy diameters, 5. the guy attachments with the tower, and 6. the areas of cross-section of all the primary and secondary members comprising the tower shaft. The limitations on maximum stresses in the members of the tower and the guys as well as the maximum deflection of the tower form the natural behaviour constraints on the design. Limitations arising due to the available guy diameters and available sections for tower members form the side constraints.

The maximum stresses in the guys and tower members and the maximum deflection of the guyed tower are dependent on the relative stiffnesses of the tower shaft and that of the guys. As the guys are pre-tensioned, the stiffness of the guys is a function of the amount of the initial tension besides the guy diameters and the slope of the guy chords.

Thus the choice of the initial guy tension, the guy diameter as well as the slope of the guy chords are obvious design variables. The location of the joints between the guys and the tower shaft is another important consideration for the design of the guyed tower.

Considering a design variable corresponding to each member of the tower, results into a large number of design variables. For example, a 100 m high tower will have about 500 members. One way to reduce this number is to select three design variables per span; span of the tower being defined as the distance between the end of the tower and the nearest guy level or the distance between two consecutive guy levels. These three design variables correspond to the areas of cross-section of the leg member, the diagonal bracing member on the face of the tower and the web bracing member.

If the number of guy levels is also considered as a design variable, the problem turns out to be a mixed integer programming problem with large number of variables. Also, one has to note that the areas of cross-section of the members of the tower and the diameters of the guys are available only in discrete quantities.

As is clear from the above qualitative discussion, the resulting optimum design problem will be a mixed integer programming problem with a large number of design variables and larger number of constraints. A direct solution of such a large integer programming

problem is as of today intractable. Therefore, it is natural to look for the possible reduction in the size of the problem. Some of the parameters can be suitably predefined thus reducing the number of design variables. The number of guy levels is directly dependent upon the height of the guyed tower. The tall guyed towers in use vary from 100 m to 400 m. The variation in the number of guy levels for the range of height mentioned is 3 to 7. The larger the number of guy levels for a particular tower, the more would be the ground space covered by the tower. From structural point of view, a particular tower with more number of guy levels will experience more axial thrust, thus undergoing more destabilizing effect. However, increased number of guy levels shall contribute to the over-all flexural stiffness of the guyed tower. Therefore, fixing the number of guy levels for a particular tower is in itself a subject matter of study to be approached from the over-all stability and flexural stiffness considerations. This, however, has not been studied in the present work and based on the experience of the earlier investigators, the number of guy levels has been fixed in the present work.

The stresses in the bracing members are observed to be very nearly the same in all the spans. Thus, it is reasonable to assume that all the web braces have the same area of cross-section and similarly all the diagonal members will have the same area of cross-section, throughout the height of the tower.

The problem can be further simplified by treating the design variables corresponding to the areas of cross-section of the members of the tower and the diameters of the guys as continuous variables, though it is not so in actual practice. Thus the simplified optimum design problem for a guyed tower having m guy levels will have $5m + 3$ or $5m + 2$ design variables, depending upon whether the tower has a cantilever portion at the top or not.

As the effect of large deformation of the tower on the performance of the guyed tower is found to be only marginal [16], it has not been considered in the present work. The analysis procedure used in the present work is discussed in [17], and for completeness of presentation, is described in Chapter 2. The guys are treated as parabolas. The tower shaft is analysed as a beam-column resting on elastic supports. Only loads due

to wind action are considered in the present work. The strengths of the tower members are calculated as per the Indian Standard Specifications [12].

The optimum design problem is formulated in Chapter III, where in, the method of solution is briefly described. The results of sample problems are given in Chapter IV. The conclusions of the present work are also summarised in this chapter alongwith the further scope of work.

The computer programs developed and used in the present work are briefly described in Appendix-A. The listing of these programs and the input data required are given in Appendices B and C respectively.

CHAPTER II

METHOD OF ANALYSIS

2.1 INTRODUCTION

The guyed tower is a highly non-linear structure even under elastic conditions. The non-linearity is basically due to two reasons: 1. due to the non-linearities of the guys and 2. due to the heavy axial loads. The axial thrust is responsible for reducing the stiffness of the tower shaft. Axial thrust is partly due to the vertical components of the tensions in the guys. Thus, the two non-linearities are coupled together. An iterative approach is an obvious choice, to seek the solution of such a non-linear structure. Moreover, the method of analysis has to be simple and accurate because, it is to be embedded in the analysis, design cycle for seeking the optimum design. Based on these considerations, the method of analysis suggested by Odley [17] is used in the present work. This method is based on the following assumptions:

1. Wind loads on tower shaft are known and are assumed to be uniform between guy levels.

2. The moment of inertia of the tower shaft is considered to be uniform between guy levels.
3. Dead load of the tower shaft for each span is concentrated, one-half at each end of beam-column action only.
4. The guys are uniformly loaded by wind.
5. The velocity of wind acting on a guy is the wind velocity at its average height.
6. Guy curve is parabola for all loading conditions.
7. Drag and lift coefficients for a guy are as indicated by Diehl [6].
8. Wind is blowing parallel to the ground.

2.2 LOADS ON THE STRUCTURE

The tower shaft and the guys are subjected to the following loads:

2.2.1 Loads on the Guys

(i) Dead load: This includes the self weight of the guys, weight of the insulators and weight of the ice, if deposited on the guys. However, in the present work, only the self weight of the guys is considered.

(ii) Wind loads: Wind exerts forces in the direction of wind in the form of drag and in the direction perpendicular to wind, in the form of lift. Guy tensions are increased due to these forces. The calculation of drag and lift forces is dealt in a subsequent section.

2.2.2 Loads on the Tower Shaft:

(i) Dead Loads: Dead loads consist of self weight of the shaft including the antennas and other ancillaries attached to the mast; e.g. stairs, warning signals for aircrafts etc.

(ii) Live Loads:

a) Wind load: This is the primary live load which comes on the tower. The force on a tall structure like a guyed tower due to wind loading is a function of the properties of air, the wind velocity, the height under consideration and the drag of the structure. The velocity pressure for moving air is given by the following formula:

$$V_p = \frac{\sqrt{V}^2}{2g} = \frac{1.2518 V^2}{2 \times 9.81}$$

$$\text{i.e. } V_p = 0.0049 V^2 \quad (2.2.1)$$

where, V_p is the velocity pressure in kg/cm^2 ;

γ is the specific weight of air
(1.2518 kg/m³ at 15.5°C).

V is velocity of air in kmph

and g is the gravitational acceleration (9.81 m/sec²)

Wind velocity is assumed to escalate with height as per the power law.

$$V_h = V_o \left(\frac{H_z}{H_o} \right)^\alpha \quad (2.2.2)$$

where,

H_o is the base height in m;

H_z is the height under consideration in m;

V_o is the basic wind velocity in kmph;

α is the wind escalation exponent;

and V_h is the wind velocity at height H_z in kmph.

Wind velocities are considered to be constant below the base height and above a certain cut-off height, H_{max} . Generally, H_o and H_{max} are taken as 10 m and 375 m respectively.

Wind escalation exponent, α , is taken as 1/8 in the present work, as it gives nearly the same order of magnitude of wind forces as recommended by IS 875 [13].

Eq. 2.2.1 makes no allowance for the shape of the object or the drag. The drag coefficient or shape factor is a function of physical size, shape, surface roughness, orientation with the air stream, openings etc.

There are various practices for estimating shape factors. In the present work, the shape factors are calculated on the basis of solidity ratio [24].

$$\text{Solidity ratio } (\phi_s) = \frac{\text{Projected net area}}{\text{Gross area}}$$

In the present work, the solidity ratio is found to be always less than 0.4. Therefore, the plot for shape factors (based on solidity ratio) given by Sachs [24], linearized between 0.0 and 0.4 is used. The linearized model is

$$C_{Dt} = 3.5 - 4.125 \phi_s \quad (2.2.3)$$

where,

ϕ_s is the solidity ratio;

and

C_{Dt} is the drag coefficient of the tower shaft.

The unit wind pressure in kg/m^2 on the tower shaft in an air stream after considering drag coefficient, C_{Dt} , is given by

$$p = V_p \cdot C_{Dt} = 0.0049 V^2 C_{Dt} \quad (2.2.4)$$

b) Ice Loads Accretion of Ice increases the vertical load on the tower shaft and increases the tensions in the guys. It is very rare that the ice load and the worst wind conditions occur simultaneously. However, ice loads are not considered in the present work.

c) Service Loads These loads depend upon a particular tower according to its function and are defined in the specifications of the tower. Loads due to the lifts provided is an example of service loads.

d) Loads due to guy reactions The guy tensions exert loads on the tower at the guy attachments. The horizontal components of the guy reactions offer resistance to tower displacements. Their vertical components tend to increase the tower displacements as their effect is to reduce the flexural strength of the tower shaft, besides increasing its axial deformation.

2.2.3 Worst Conditions of Loading

The wind conditions which result in maximum forces in the lag members and bracing members are different [22]. The critical wind conditions for triangular guyed towers are shown in Fig. 2.1. For obtaining maximum bending moment and shear forces, in a triangular tower, it is necessary to consider corner wind and face wind conditions. Maximum leg forces are found from the consideration of these wind conditions. Although maximum bracing forces can be obtained from the side wind condition, the analysis carried out according to corner wind and Face Wind conditions is considered sufficient [22].

Maximum guy tensions are obtained for triangular towers with the corner wind condition as shown in Fig. 2.1.

2.3 ANALYSIS

The analysis of the guys and the tower shaft based on the assumptions enlisted in Section 2.1 and the loads described in Section 2.2 is given in this section.

2.3.1 Analysis of Guys

2.3.1.1 Normal position of guy : Fig. 2.2 shows a guy with no wind blowing and at normal temperature, to. If

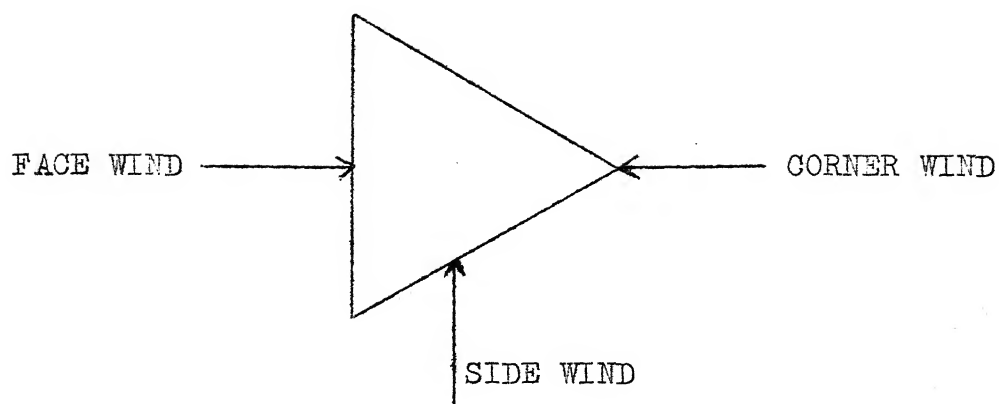


FIG. 2.1 CRITICAL WIND CONDITIONS FOR GUYED TOWERS TRIANGULAR IN SHAPE

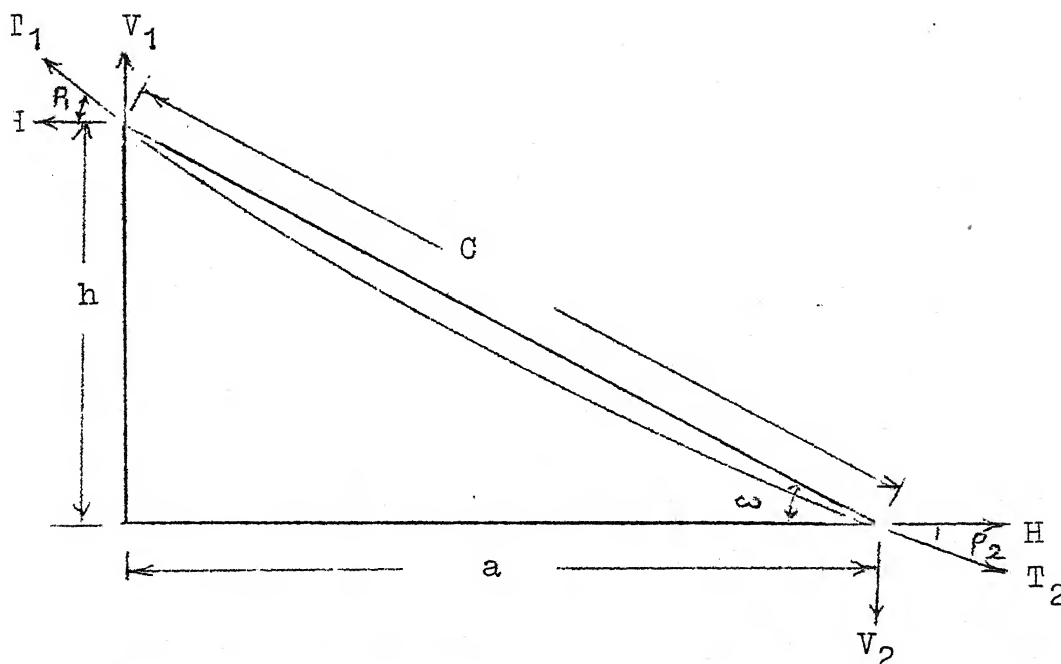


FIG. 2.2 ELEVATION OF A GUY IN NORMAL POSITION

the total weight of the guy is W tons, the approximate length of the guy L , in meters is given by

$$L = a \left(\sec \omega + \frac{W^2}{24 H^2 \sec^3 \omega} \right) \quad (2.3.1)$$

where

H is the horizontal component of the guy tension in tons.

For erection purposes, the tension at the anchorage, T_2 , is known and H can be computed from

$$H = \frac{\cos \omega}{2} \left[W \sin \omega + \sqrt{(4T_2^2 - W^2 \cos^2 \omega)} \right] \quad (2.3.2)$$

It may be shown for a parabola that

$$\tan \phi_1 = \tan \omega + \frac{W}{2H} \quad (2.3.3a)$$

$$\tan \phi_2 = \tan \omega - \frac{W}{2H} \quad (2.3.3b)$$

and

$$\Delta_g = \frac{H a}{A_g E_g} \left(\sec^2 \omega + \frac{W^2}{12 H^2} \right) \quad (2.3.3c)$$

where,

Δ_g is the stretch of guy in meters;

A_g is metallic area of guy in sq.cm.;

and

E_g is modulus of elasticity of guy in tons/sq.cm.

The unstressed length of the guy, L_0 , at normal temperature, t_0 , is then

$$L_0 = L - \Delta_g \quad (2.3.4)$$

At a temperature t degrees Fahrenheit, the unstressed length, L_t , is given by

$$L_t = L_0 [1 + 0.0000065 (t - t_0)] \quad (2.3.5)$$

2.3.1.2 Effect of tower Motion and Wind on Guy: The forces acting on the guy are shown in Fig. 2.3. The projection of guy chord on a horizontal plane makes an angle ϕ with the direction of wind. Point A represents the point of attachment to the tower which, in this case, is assumed not to have moved. Positive directions of the forces are shown in Fig. 2.3. The total weight, W , acts vertically down; the total drag, d_0 , acts parallel to the Y-axis; and the lift, l_1 , is parallel to AD and normal to d_0 . θ is the true angle between the guy chord and the direction of wind is given by

$$\cos \theta = \cos \phi \cos \omega \quad (2.3.6)$$

Let l and l_h represent the components of the lift, l_1 , in the Z and X directions respectively; then

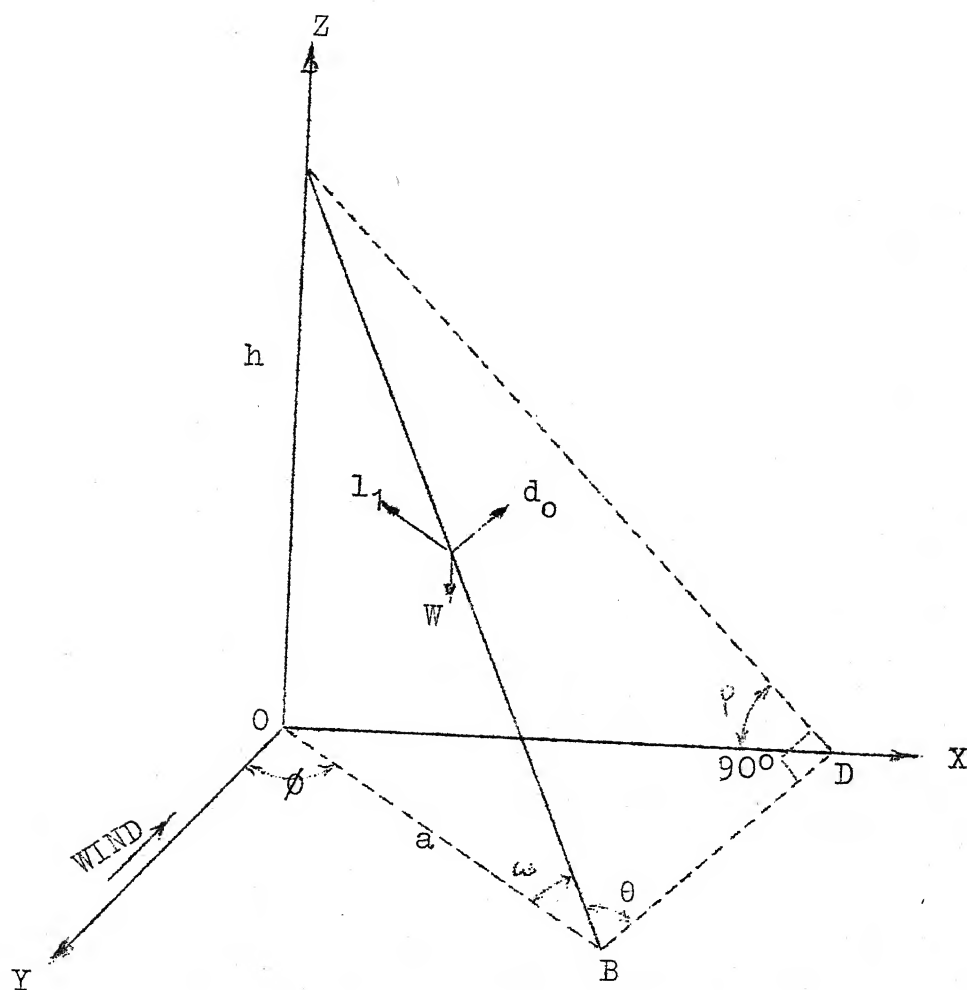


FIG. 2.3 LOADS ON GUY

$$l = l_1 \sin \rho \quad (2.3.7a)$$

$$l_h = l_1 \cos \rho \quad (2.3.7b)$$

in which,

$$\sin \rho = \frac{\sin \omega}{\sin \theta}; \quad (2.3.8a)$$

and

$$\cos \rho = \frac{\sin \phi \cos \omega}{\sin \theta} \quad (2.3.8b)$$

Assuming the velocity pressure to be given by Eq. 2.2.1, the total drag, d_o , and lift, l_1 , on a guy are approximately,

$$d_o = 0.0049 C d V^2 C_D 10^{-4} \text{ tons} \quad (2.3.9)$$

and

$$l_1 = 0.0049 C d V^2 C_L 10^{-4} \text{ tons} \quad (2.3.10)$$

where,

C is the chord length of guy in m;

d is the diameter of guy in cm;

V is the wind velocity in kmph;

C_D is the drag coefficient;

and

C_L is the lift coefficient.

It should be noted that l_1 is negative when ϕ is in the first or fourth quadrant. Hence, the sign of l_1 is opposite to that of $\cos \phi$.

Diehl [6] indicated values of C_D and C_L by curves for values of θ varying from 0° to 90° . These values can be expressed in polynomial form as

$$C_D = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 \quad (2.3.11a)$$

$$C_L = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 \quad (2.3.11b)$$

where,

$$x = |\cos \theta|$$

Depending upon the direction of wind, the values constants in polynomials represented by Eqs. 2.3.11a and 2.3.11b are given in tables 2.1 and 2.2.

TABLE 2.1 : Values of Constants in Polynomial, Eq.2.3.11a,
for C_D

x	a_0	a_1	a_2	a_3	a_4
$x \leq 0.575$	1.18457	0.07816	-1.51543	-1.73395	2.56634
$x > 0.575$	1.20931	0.08774	-1.33619	-0.82684	1.06662

TABLE 2.2 : Values of Constants in Polynomial, Eq.2.3.11b,
for C_L

x	a_0	a_1	a_2	a_3	a_4
$x \leq 0.575$	0.00008	1.45668	-2.73481	5.36663	-4.75092
$x > 0.575$	1.40075	-4.20644	6.00561	-2.24738	-0.94991

In Fig. 2.4, the guy has deflected a distance Δ meters in the direction of wind at the guy attachment point. Due to the wind loads, the guy lies in a new plane that contains the resultant of all the forces acting on the guy, W_r . This new guy plane is represented by A_1O_1B in the figure. The guy is assumed to be a parabola lying in this plane with A_1O_1 parallel to W_r and O_1B normal to W_r . W_r is given by

$$W_r = \sqrt{[l_h^2 + d_o^2 + (W-1)^2]} \quad (2.3.12)$$

Also,

$$a_2 = \sqrt{[(a \sin \phi)^2 + (a \cos \phi + \Delta)^2]} \quad (2.3.13a)$$

$$\sin \phi_1 = \frac{a \sin \phi}{a_2} \quad (2.3.13b)$$

$$\text{and } \cos \phi_1 = \frac{a \cos \phi + \Delta}{a_2} \quad (2.3.13c)$$

It can be shown that

$$h_1 = \frac{(W-1)h_t + a_2 (l_h \sin \phi_1 + d_o \cos \phi_1)}{W_r} \quad (2.3.14a)$$

$$a_1 = \sqrt{(a_2^2 + h_t^2 - h_1^2)} \quad (2.3.14b)$$

$$\text{and } \tan \omega_1 = \frac{h_1}{a_1} \quad (2.3.14c)$$

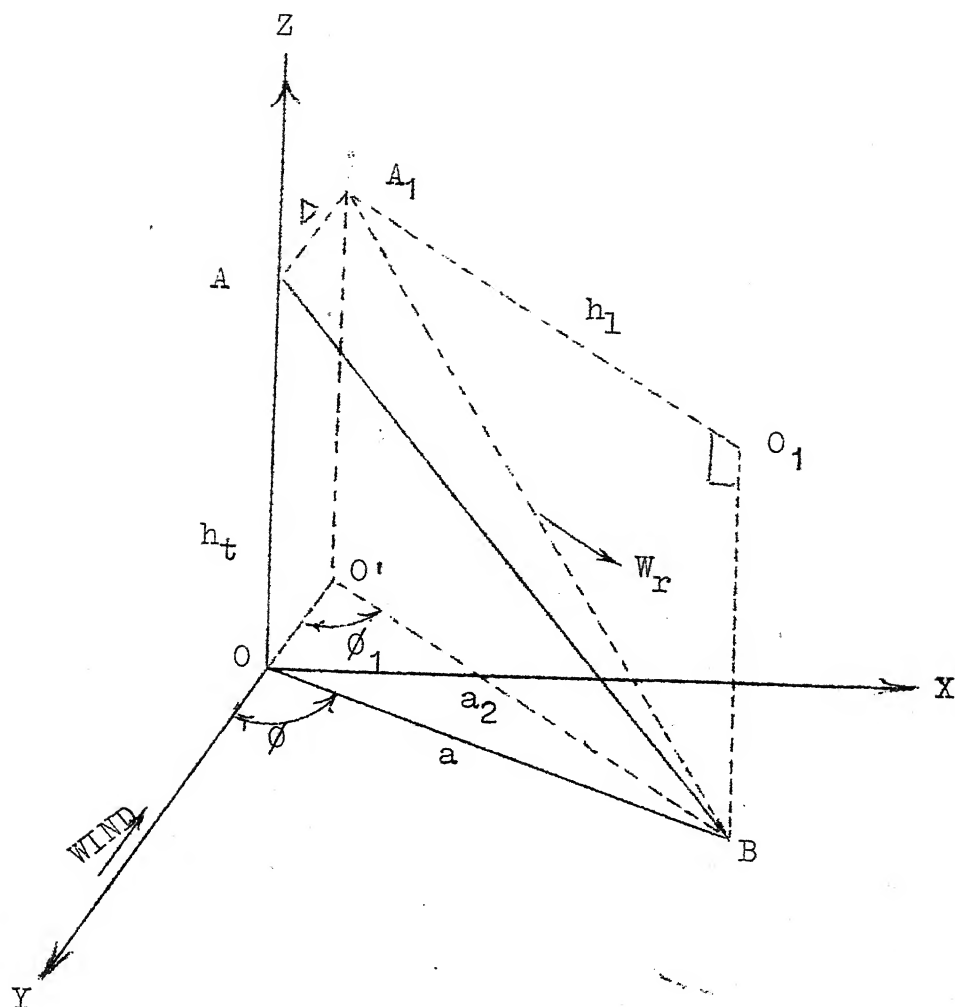


FIG. 2.4 GEOMETRY OF GUY IN DEFLECTED POSITION

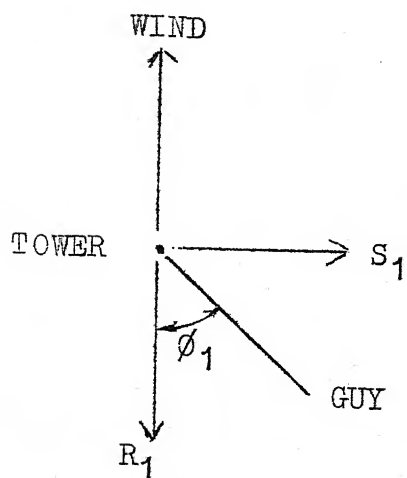


FIG. 2.5a GUY FORCES AT TOWER

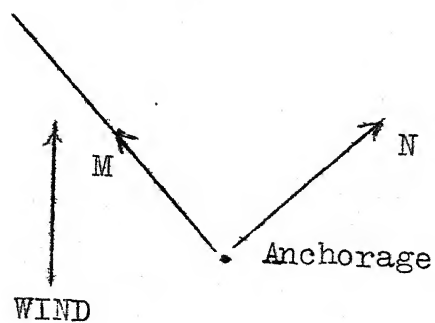


FIG. 2.5b GUY FORCES AT ANCHORAGE

in which h_t is the height of the guy at temperature, t .

The value of h_t is given by

$$h_t = h [1.0 + 0.0000065(t-t_0)] \quad (2.3.15)$$

for steel structures.

Where, h is the height of guy at normal temperature, t_0 .

The direction cosines of lines $A_1 O_1$ and $O_1 B$ are

$$\begin{array}{ll} \text{A}_1 \text{ O}_1 & \text{O}_1 \text{ B} \\ \cos \alpha_1 = -\frac{l_h}{W_r} & \cos \alpha_2 = -\tan \omega_1 \cos \alpha_1 + \frac{a_2}{a_1} \sin \phi_1 \\ \cos \beta_1 = -\frac{d_o}{W_r} & \cos \beta_2 = -\tan \omega_1 \cos \beta_1 + \frac{a_2}{a_1} \cos \phi_1 \quad (2.3.16) \\ \cos \gamma_1 = -\frac{W - l}{W_r} & \cos \gamma_2 = -\tan \omega_1 \cos \gamma_1 - \frac{h_t}{a_1} \end{array}$$

Fig. 2.5a shows the guy forces at the tower produced by the guy under wind load and Fig. 2.5b shows the forces at the anchorage. At the tower, the vertical load produced by the guy is Z_1 and the tension in the guy is T_3 . At the anchorage, the vertical uplift is Z_2 and the tension is T_4 .

If H_1 is the tension in the guy of Fig. 2.4 comparable to H of Fig. 2.2, it is possible to write:

$$\tan \phi'_1 = \tan \omega_1 + \frac{W_r}{2H_1} \quad (2.3.17a)$$

$$\tan \phi'_2 = \tan \omega_1 - \frac{W_r}{2H_1} \quad (2.3.17b)$$

$$V'_1 = H_1 \tan \phi'_1 \quad (2.3.17c)$$

$$V'_2 = H_1 \tan \phi'_2 \quad (2.3.17d)$$

$$T_3 = H_1 \sec \phi'_1 \quad (2.3.17e)$$

$$T_4 = H_1 \sec \phi'_2 \quad (2.3.17f)$$

$$R_1 = H_1 \cos \beta_2 + V'_1 \cos \beta_1 \quad (2.3.17g)$$

$$S_1 = H_1 \cos \alpha_2 + V'_1 \cos \alpha_1 \quad (2.3.17h)$$

$$Z_1 = -H_1 \cos \gamma_2 - V'_1 \cos \gamma_1 \quad (2.3.17i)$$

$$M = H_1 (\sin \phi_1 \cos \alpha_2 + \cos \phi_1 \cos \beta_2) \\ + V'_2 (\sin \phi_1 \cos \alpha_1 + \cos \phi_1 \cos \beta_1) \quad (2.3.17j)$$

$$N = H_1 (\sin \phi_1 \cos \beta_2 - \cos \phi_1 \cos \alpha_2) \\ + V'_1 (\sin \phi_1 \cos \beta_1 - \cos \phi_1 \cos \alpha_1) \quad (2.3.17k)$$

$$\text{and } Z_2 = -H_1 \cos \gamma_2 - V'_2 \cos \gamma_1 \quad (2.3.17l)$$

where V'_1 is parallel to W_r and H_1 is normal to it.

For any particular value of Δ and temperature, t , the value of H_1 is determined by trial as follows:

1. Assume a value of H_1
2. Compute $L_1 = a_1 \left(\sec \omega_1 + \frac{W_r^2}{24 H_1^2 \sec^3 \omega_1} \right)$ (2.3.18)
3. Compute $\Delta_g' = \frac{H_1 a_1}{A_g E_g} \left(\sec^2 \omega_1 + \frac{W_r^2}{12 H_1^2} \right)$ (2.3.19a)
4. Compute $L_t' = L_1 - \Delta_g'$ (2.3.19b)
5. Compare the value of L_t' given by Eq. 2.3.19b with that given by Eq. 2.3.5. These two values should be the same, because the unstressed length of the guy is constant. If the values do not agree, the process has to be repeated with a new value of H_1 , until satisfactory agreement is reached.

Having obtained H_1 , forces at the tower, anchorage and the guy tensions may be found from Eqs. 2.3.17c to 2.3.17l.

The same procedure is applied to all the guys at a level. Having determined the forces at the tower for all guys, resultants can be obtained by superposition. Thus,

$$R = \sum R_1 \quad (2.3.20)$$

and
$$Z = \sum Z_1 \quad (2.3.21)$$

where, R is the net guy reaction in a direction opposed to the wind at the guy level in consideration;
and Z is the total vertical, downward load produced by the guys, at the tower.

Because the resultant, Z , of the Z_1 forces is eccentric to the centroid of the tower shaft, an external moment, \bar{M} will be introduced at the guy level given by

$$\bar{M} = \sum Z_1 e \quad (2.3.22)$$

where, e is the distance of the application of Z_1 to the centroid of the tower shaft in the direction of the wind.

Because the actual value of Δ is unknown at this time, a procedure must be used which covers the probable range of deflection values of the tower at each guy level. For computer usage, it is expedient to select an initial or lowest value for Δ at each guy level and to increment this value several times until the range of values will probably include the actual value of Δ . With experience, the range can be reduced to shorten the required computer time. For each value of Δ , the corresponding values R , Z and \bar{M} are computed.

If it is assumed that the change in R is linear for two successive values of Δ , then

$$R = K \Delta + Q \quad (2.3.23)$$

for the selected interval. The values of K and Q are simply computed from the values of R at the lower and upper bounds of the Δ interval. Thus the two constants, K and Q, associated with each value of Δ define the value of R for that interval. Because the slope of R- Δ curve change from interval to interval, values of K and Q vary with Δ . Similarly we can write

$$\tilde{M} = B \Delta + E \quad (2.3.24)$$

and
$$Z = O \Delta + J \quad (2.3.25)$$

The computer program for guy analysis derives not only R, \tilde{M} and Z values for each value of Δ , but also values of K, Q, B, E, O and J associated with the upper bound of each interval for each guy level. When numbering the guy levels, it is expedient to begin at the lowest guy level. Thus,

$$R_1 = K_1 \Delta_1 + Q_1$$

Would define the guy reaction at the first guy level for the selected interval of Δ .

2.3.2 Analysis of Tower:

Fig. 2.6 shows the forces acting on two continuous spans of a multilevel guyed tower. In the figure,

W_n and W_{n+1} are uniformly distributed loads;

M_{n-1} , M'_n , M_n and M'_{n+1} are the internal resisting moments;

\bar{M}_n is the external moment produced by the guys;

P_{n+1} is the force acting down above guy level $n+1$ including the vertical Z loads at this guy level;

and

P_n is the similar load above guy level n .

Positive directions of the loads and moments are shown.

The continuity equation at a joint n is given by [26]

$$\begin{aligned} & M'_n \frac{l_n}{3EI_n} \psi(u_n) + M_{n-1} \frac{l_n}{6EI_n} \phi(u_n) \\ & + W_n \frac{l_n^2}{24EI_n} \chi(u_n) - \frac{\delta'_n}{l_n} + M_n \frac{l_{n+1}}{3EI_{n+1}} \psi(u_{n+1}) \\ & + M'_{n+1} \frac{l_{n+1}}{6EI_{n+1}} \phi(u_{n+1}) + W_{n+1} \frac{l_{n+1}^2}{24EI_{n+1}} \chi(u_{n+1}) + \frac{\delta'_{n+1}}{l_{n+1}} = 0 \end{aligned}$$

(2.3.26)

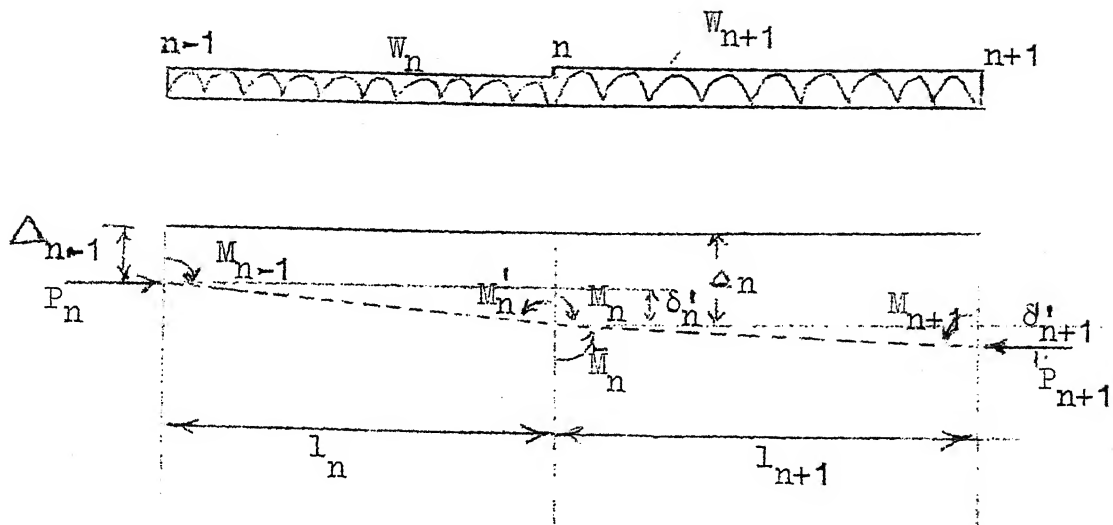


FIG. 2.6 FORCES ON TWO CONTINUOUS SPANS OF TOWER SHAFT

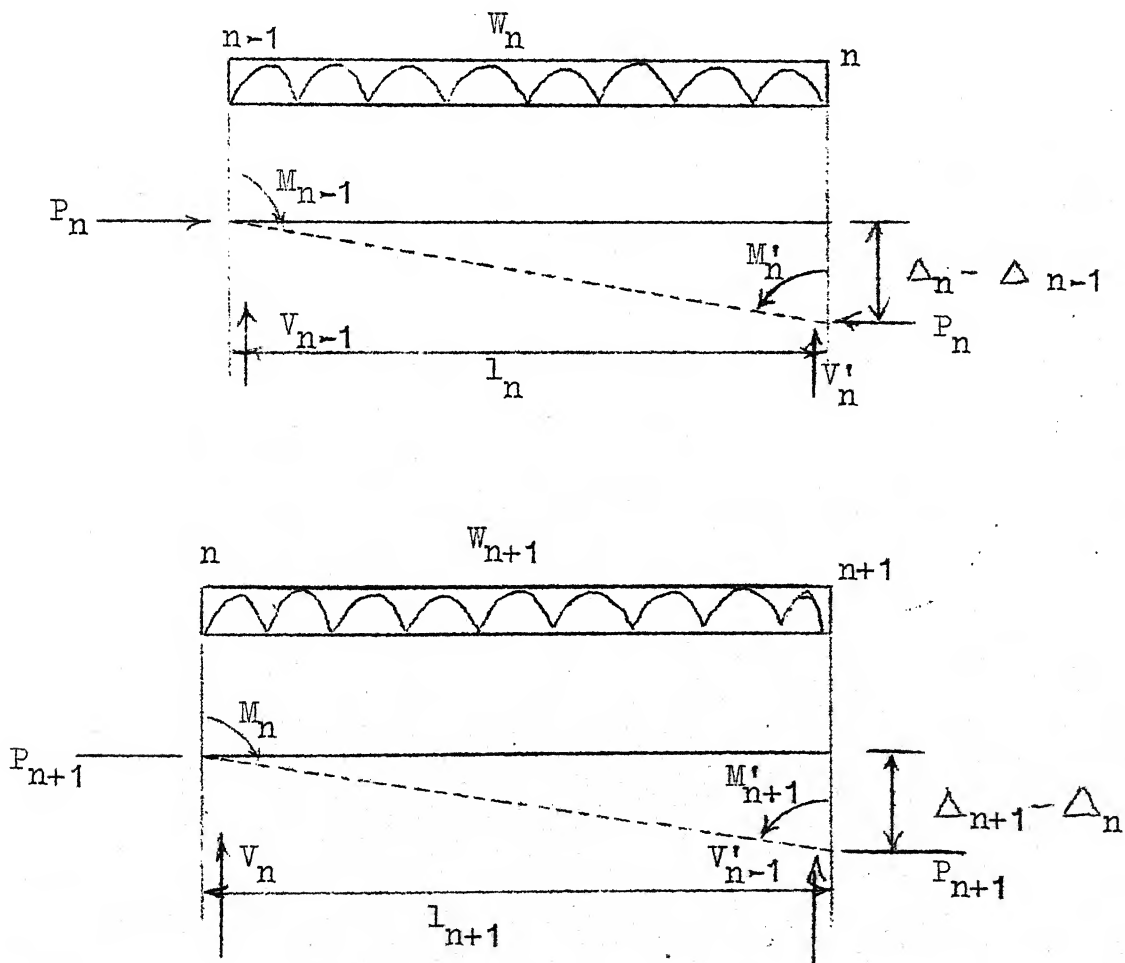


FIG. 2.7 FREE BODY DIAGRAMS OF TWO CONTINUOUS SPANS OF TOWER SHAFT

where,

I_n, I_{n+1} are the moment of inertia of the beams
of span n and $n+1$;

E is the modulus of elasticity of the tower shaft;

$$u = \frac{1}{2} \sqrt{\left(\frac{P}{EI}\right)} \quad (2.3.27)$$

$$\phi(u) = \frac{3}{u} \left(\frac{1}{\sin 2u} - \frac{1}{2u} \right) \quad (2.3.28)$$

$$\psi(u) = \frac{3}{2u} \left(\frac{1}{2u} - \frac{1}{\tan 2u} \right) \quad (2.3.29)$$

$$\text{and } \chi(u) = \frac{3(\tan u - u)}{3} \quad (2.3.30)$$

Because

$$M'_n = M_n + \bar{M}_n, \quad (2.3.31a)$$

$$M'_{n+1} = M_{n+1} + \bar{M}_{n+1}, \quad (2.3.31b)$$

$$\delta'_n = \Delta_n - \Delta_{n-1}, \quad (2.3.31c)$$

and

$$\delta'_{n+1} = \Delta_{n+1} - \Delta_n, \quad (2.3.31d)$$

Eq. 2.3.26 may be written

$$\begin{aligned}
& 4M_{n-1} \frac{l_n}{I_n} \phi(u_n) + 8M_n \left[\frac{l_n}{I_n} \psi(u_n) + \frac{l_{n+1}}{I_{n+1}} \psi(u_{n+1}) \right] \\
& + 4(M_{n+1} + \bar{M}_{n+1}) \frac{l_{n+1}}{I_{n+1}} \phi(u_{n+1}) + 8\bar{M}_n \frac{l_n}{I_n} \psi(u_n) \\
& + W_n \frac{l_n^2}{I_n} \chi(u_n) + W_{n+1} \frac{l_{n+1}^2}{I_{n+1}} \chi(u_{n+1}) \\
& = 24E \frac{\Delta_n - \Delta_{n-1}}{l_n} - \frac{\Delta_{n+1} - \Delta_n}{l_{n+1}} \quad (2.3.32)
\end{aligned}$$

Eq. 2.3.37 is the typical continuity equation.

Fig. 2.7 shows free body diagrams of the two spans of Fig. 2.6 with applied forces as indicated.

For equilibrium, V'_n and V_n are given by

$$V'_n = \frac{W_n}{2} + \frac{M_{n-1} - M'_n}{l_n} + \frac{P_n}{l_n} (\Delta_n - \Delta_{n-1}) \quad (2.3.33a)$$

$$V_n = \frac{W_{n+1}}{2} + \frac{M'_{n+1} - M_n}{l_{n+1}} - \frac{P_{n+1}}{l_{n+1}} (\Delta_{n+1} - \Delta_n) \quad (2.3.33b)$$

Because $R_n = V'_n + V_n$, using Eqs. 2.3.31a, 2.3.31b, 2.3.33a and 2.3.33b, the value of R_n is given by

$$\begin{aligned}
 R_n = & \frac{1}{2} (W_n + W_{n+1}) + \frac{M_{n-1}}{l_n} - M_n \left(\frac{1}{l_n} + \frac{1}{l_{n+1}} \right) + \frac{M_{n+1}}{l_{n+1}} \\
 & - \frac{\bar{M}_n}{l_n} + \frac{\bar{M}_{n+1}}{l_{n+1}} + \frac{P_n}{l_n} (\Delta_n - \Delta_{n-1}) - \frac{P_{n+1}}{l_{n+1}} (\Delta_{n+1} - \Delta_n)
 \end{aligned}
 \tag{2.3.34}$$

Eq. 2.3.34 is the typical interior reaction equation.

For a tower with m spans, the continuity and reaction equations, Eqs. 2.3.32 and 2.3.34, must be modified for the end spans. For continuity at support 1, Eq. 2.3.32 becomes,

$$\begin{aligned}
 4M_0 \frac{l_1}{I_1} \phi(u_1) + 8M_1 \left[\frac{l_1}{I_1} \psi(u_1) + \frac{l_2}{I_2} \psi(u_2) \right] \\
 + 4(M_2 - \bar{M}_2) \frac{l_2}{I_2} \phi(u_2) + 8\bar{M}_1 \frac{l_1}{I_1} \psi(u_1) + W_1 \frac{l_1^2}{I_1} \chi(u_1) \\
 + W_2 \frac{l_2}{I_2} \chi(u_2) = 24E \left[\frac{\Delta_1}{l_1} - \frac{\Delta_2 - \Delta_1}{l_2} \right]
 \end{aligned}
 \tag{2.3.35}$$

and at support $m-1$, the term M_{m+1} is omitted. For the reaction at the first support, Eq. 2.3.34 becomes,

$$\begin{aligned}
 R_1 = & \frac{1}{2} (W_1 + W_2) + \frac{M_0}{l_1} - M_1 \left(\frac{1}{l_1} + \frac{1}{l_2} \right) + \frac{M_2}{l_2} \\
 & - \frac{\bar{M}_1}{l_1} + \frac{\bar{M}_2}{l_2} + \frac{P_1}{l_1} \Delta_1 - \frac{P_2}{l_2} (\Delta_2 - \Delta_1)
 \end{aligned}
 \tag{2.3.36}$$

and for the section at m ,

$$R_m = \frac{1}{2} W_m + \frac{M_{m-1}}{l_m} - \frac{\bar{M}_m}{l_m} + \frac{P_m}{l_m} (\Delta_m - \Delta_{m-1}) \quad (2.3.37)$$

If the tower is hinged at the base, $M_0 = 0$ in the above equations. If the base is fixed, there is no angle change and,

$$\begin{aligned} M_0 \frac{l_1}{3EI_1} \psi(u_1) + M_1 \frac{l_1}{6EI_1} \phi(u_1) \\ + W_1 \frac{l_1^2}{24EI_1} \chi(u_1) + \frac{\Delta_1}{l_1} = 0 \end{aligned} \quad (2.3.38a)$$

or

$$\begin{aligned} 8M_0 \frac{l_1}{I_1} \psi(u_1) + 4(M_1 + \bar{M}_1) \frac{l_1}{I_1} \phi(u_1) \\ + W_1 \frac{l_1^2}{I_1} \chi(u_1) + 24E \frac{\Delta_1}{l_1} = 0 \end{aligned} \quad (2.3.38b)$$

For a tower with m guy levels, $2m$ equations can be written for a fixed base and $2m-1$ equations can be written for a hinged base. The unknowns in these equations are M_0, M_1, \dots, M_{m-1} and R_1, R_2, \dots, R_m .

The above equations contain the values of the tower deflections Δ_1, Δ_2 etc., which are also unknown, thereby increasing the number of unknowns to $3m$ for a

fixed base and $3m-1$ for a hinged base. Using the constants K , Q , B , E , O and J , the Δ values can be eliminated, thus reducing the number of unknowns to the same number of available equations.

From Eqs. 2.3.23 to 2.4.25

$$\Delta_i = \frac{R_i - Q_i}{K_i}, \quad i = 1, 2, \dots, m \quad (2.3.39a)$$

$$\bar{M}_i = \frac{B_i}{K_i} (R_i - Q_i) + E_i, \quad i = 1, 2, \dots, m \quad (2.3.39b)$$

where

Δ_i is the deflection at i th guy level.

Using 2.3.39a, 2.3.39b, 2.3.31a and 2.3.31b the general continuity equation, Eq. 2.3.32 can be written as:

$$\begin{aligned} & 4 M_{n-1} \frac{l_n}{I_n} \phi(u_n) + 8 M_n \left[\frac{l_n}{I_n} \psi(u_n) \frac{l_{n+1}}{I_{n+1}} \psi(u_{n+1}) \right] \\ & + 4 \frac{l_{n+1}}{I_{n+1}} \phi(u_{n+1}) \left[M_{n+1} + \frac{B_{n+1}}{K_{n+1}} (R_{n+1} - Q_{n+1}) + E_{n+1} \right] \\ & + 8 \frac{l_n}{I_n} \psi(u_n) \left[\frac{B_n}{K_n} (R_n - Q_n) + E_n \right] + W_n \frac{l_n^2}{I_n} \chi(u_n) \\ & + W_{n+1} \frac{l_{n+1}^2}{I_{n+1}} \chi(u_{n+1}) = 24E \left[\left(\frac{1}{l_n} + \frac{1}{l_{n+1}} \right) \left(\frac{R_n - Q_n}{K_n} \right) \right. \\ & \quad \left. - \frac{1}{l_n} \left(\frac{R_{n-1} - Q_{n-1}}{K_{n-1}} \right) - \frac{1}{l_{n+1}} \left(\frac{R_{n+1} - Q_{n+1}}{K_{n+1}} \right) \right] \end{aligned} \quad (2.3.40)$$

and the general interior reaction equation Eq. 2.3.34 becomes

$$\begin{aligned}
 R_n = & \frac{1}{2} (W_n + W_{n+1}) - M_n \left(\frac{1}{l_n} + \frac{1}{l_{n+1}} \right) + \frac{M_{n-1}}{l_n} + \frac{M_{n+1}}{l_{n+1}} \\
 & - \frac{1}{l_n} \left[\frac{B_n}{K_n} (R_n - Q_n) + E_n \right] + \frac{1}{l_{n+1}} \left[\frac{B_{n+1}}{K_{n+1}} (R_{n+1} - Q_{n+1}) + E_{n+1} \right] \\
 & + \left[\frac{P_n}{l_n} + \frac{P_{n+1}}{l_{n+1}} \right] \frac{(R_n - Q_n)}{K_n} - \frac{P_n}{l_n} \left[\frac{R_{n-1} - Q_{n-1}}{K_{n-1}} \right] - \frac{P_{n+1}}{l_{n+1}} \left[\frac{R_{n+1} - Q_{n+1}}{K_{n+1}} \right]
 \end{aligned}
 \tag{2.3.41}$$

Guy constants O_1 , J_1 , etc. are used to determine the values of P_1 , P_2 , etc. In a similar manner, values of Δ can be eliminated from the end span equations.

In many cases, the span above the top guy is cantilevered and in some instances, the cantilever is loaded with an antenna pull-off with an additional moment and vertical load applied at the top of the tower. Assuming a cantilever above the m^{th} guy level, as in Fig. 2.8, and neglecting the effect of the moments caused by the force T in span l_c , the value of Δ_c is given by

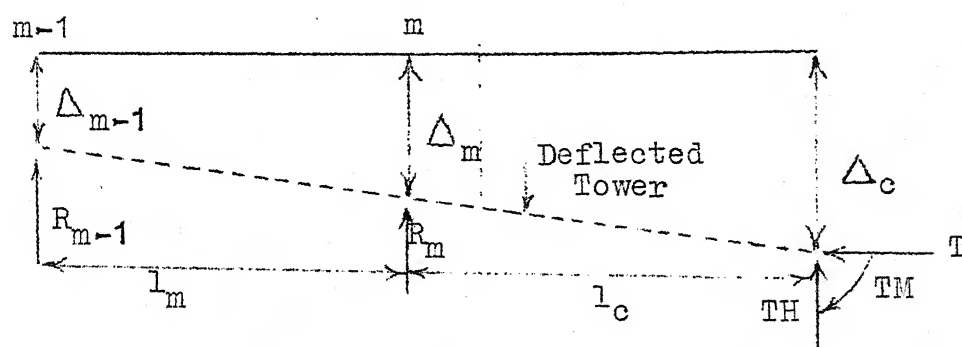
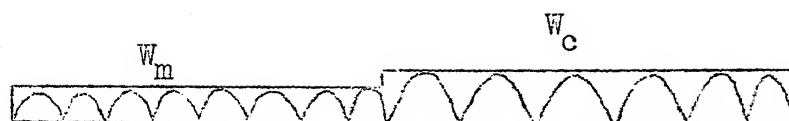


FIG. 2.8 LOADS ON CANTILEVERED SPAN OF TOWER

$$\Delta_c = \Delta_m + \frac{1}{EI_c} \left[\frac{TH l_c^3}{3} + \frac{W_c l_c^3}{8} + \frac{TM l_c^2}{2} \right] - l_c \theta_m \quad (2.3.42)$$

where,

θ_m is the angle change at m^{th} level;

and I_c is the moment of inertia of the cantilevered span.

In the usual design cases, the effect of θ_m is negligible and hence θ_m can be considered to be zero.

For $\theta_m = 0$, Eq. 2.3.42 becomes

$$\Delta_c = \Delta_m + \xi \quad (2.3.43)$$

in which

$$\xi = \frac{l_c^2}{24EI_c} \left[l_c (8TH + 3W_c) + 12 TM \right] \quad (2.3.44)$$

The cantilever moment, G , at m is given by

$$G = W_c \frac{l_c}{2} + TH l_c + TM + \xi (T + 0.5 D_c) \quad (2.3.45)$$

where D_c is the total weight of the cantilever.

Having determined the reactions, the deflections and remaining moments can be computed by means of Eqs. 2.3.23, 2.3.24 and 2.3.31a to 2.3.31d. Although the work of establishing and solving them is involved, it is accomplished in rather a short time by the use of computer.

It is necessary to compute the spring constants at the various guy levels before the tower is analysed. The tower analysis is made by first assuming deflections of the tower shaft at each guy level. For these assumed deflections, the constants, K , Q , B , E , O , and J for each guy level are known and the necessary simultaneous equations are established and solved to obtain moments and reactions. Having determined the reactions, the resulting deflections are computed from Eqs. 2.3.39a and 2.3.39b. These deflections are taken as the new assumed values and this process is repeated until convergence is achieved.

2.3.2.1 Two way bending of tower: In cases, where the direction of antenna pull-off is different from that of the wind, it is expedient to resolve the pull-off load and moment at the top of the tower into two components, one parallel to the wind and the other normal to the direction of wind. If the bending in each direction is computed with the same vertical loads, P , the results may be combined by superposition.

Because the guy constants computed correspond to the motion in the direction of the wind only, it becomes necessary to modify the equations to include tower motion in two directions and to develop a technique for finding the true deflections of the tower at each guy level.

As before, the deflection of tower in the direction of wind is taken as Δ and deflection normal to it is δ , as indicated in Fig. 2.9.

Considering the tower deflections to be positive as indicated, only two changes are required in the guy equations. Eq. 2.3.13a becomes

$$a_2 = \sqrt{[(a \sin \phi - \delta)^2 + (a \cos \phi + \Delta)^2]} \quad (2.3.46)$$

and Eq. 2.3.13b becomes

$$\sin \phi_1 = \frac{a \sin \phi - \delta}{a_2} \quad (2.3.47)$$

in which the positive direction of S_1 is as shown in Fig. 2.5a.

As there are deflections of tower in two directions at each guy level, the problem of finding these deflections by trial is more involved than in the previous case. Assuming pull-off as shown in Fig. 2.10, the following

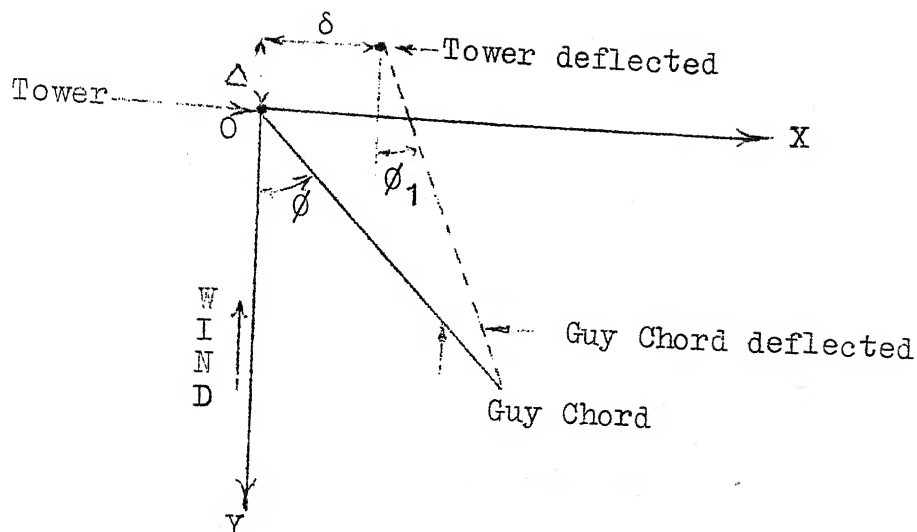


FIG. 2.9 GEOMETRY OF TWO-WAY MOTION OF TOWER AT GUY CONNECTION

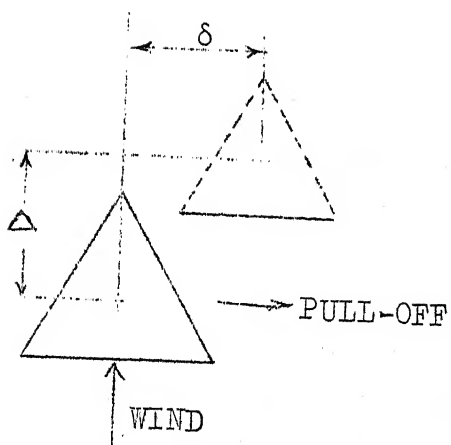


FIG. 2.10 DEFLECTIONS OF TOWER SHAFT AT GUY CONNECTION

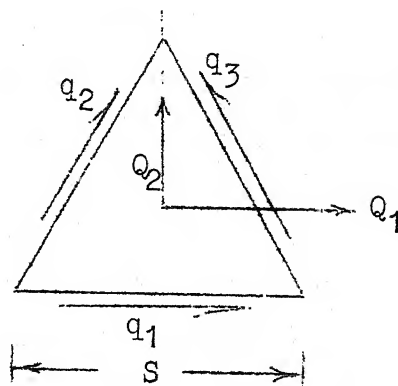


FIG. 2.11 SHEAR FORCES ON SECTION OF TRIANGULAR TOWER SHAFT

procedure |17| is used in the present work.

1. Assume Δ values for each level.
2. Compute guy constants K, Q, B, E, O, and J for various values of δ at each guy level keeping the values of Δ in step 1 constant.
3. Assume values of δ at each level and solve for actual values. For this, assume no wind loads on the tower. Use guy constants in the δ direction for this computation.
4. Repeat step 3 till the values of δ converge at each guy level.
5. With the values of δ computed in step 4 constant, Compute guy constants for various values of Δ at each guy level.
6. With the constants of step 5, solve for tower deflections until assumed and computed values are in substantial agreement.
7. Repeat the process until all deflections converge.

Having determined the reactions and moments in both directions, it becomes a simple matter to compute the member stresses. In computing web member stresses for two way bending of triangular towers, however, it is

necessary first to determine the shear flow in each face.

Fig. 2.11 shows a section of a triangular tower acted upon by shears Q_1 and Q_2 . If the twist is zero, the shear forces in each face may be found from

$$q_1 = q_2 + q_3$$

$$(q_2 + q_3) \cos 30^\circ = Q_2 \quad (2.3.48)$$

$$(q_2 + q_3) \sin 30^\circ + q_1 = Q_1$$

Solving the above equations yields

$$q_1 = \frac{2}{3} Q_1$$

$$q_2 = \frac{1}{3} Q_1 + Q_2 \tan 30^\circ \quad (2.3.49)$$

$$q_3 = -\frac{1}{3} Q_1 + Q_2 \tan 30^\circ$$

The web stresses are then determined from q_1 , q_2 , q_3 the shear flow in each face.

CHAPTER III

OPTIMUM DESIGN PROBLEM

FORMULATION AND MINIMIZATION ALGORITHM

3.1 INTRODUCTION

When a means for predicting the behaviour of any design within a particular design concept is available, limitations on the performance and other external constraints on the design can be stated and an acceptance criterion can be established, it is possible to cast the design modification problem in the form of a mathematical programming problem.

In general a mathematical programming problem can be stated as

$$\begin{aligned} &\text{Minimize (or maximize) } f(\vec{X}) \text{ such that} \\ &g_j(\vec{X}) \leq 0, \quad j = 1, 2, \dots, l \end{aligned} \tag{3.1.1}$$

where,

\vec{X} is a n -dimensional vector of design variables;

$F(\vec{X})$ is the objective function to be optimized;

$g_j(\vec{X})$ are the constraints on the design;

and l is the total number of constraints.

In this chapter, the automated optimum design problem of multi-level guyed towers is formulated as a mathematical programming problem. The design criteria used in the present work have been discussed and the reasons for choosing the cost as the objective function have been described. The reasons for selecting the design variables are discussed and the equations corresponding to the constraints imposed on the automated optimum design are developed. The solution procedure adopted to seek the solution is also briefly discussed in this Chapter.

3.2 DESIGN CRITERIA

Characterization of a design philosophy involves many considerations. In the present work, the problem of designing the tall guyed tower has been considered to be deterministic as against probabilistic. In other words, all the quantities involved in the analysis and the design of the structural system are considered to be deterministic. The adequate performance of the structural system has been sought by trying to avoid failure modes such as initial yielding and excessive deflection. Dynamic response characteristics are not considered in the present work. In other words, the present work

is an attempt to obtain the optimum design of a multilevel guyed tower within the deterministic static elastic regime.

3.3 OBJECTIVE FUNCTION

The nature of the structural design problem is such that there will usually be many designs that perform the specified functional purposes adequately. A basis for choice between alternate acceptable designs should be selected in order to cast an automated optimum structural design problem. Cost minimization is frequently taken to be the objective of optimization of civil engineering structures. Hence, minimum cost of tall guyed tower has been considered as the objective function in the present work.

The cost of the guyed tower can be represented by

$$f(\vec{X}) = C_t + C_g \quad (3.3.1)$$

where,

C_t is the cost of the tower shaft;

and C_g is the cost of the guys.

C_t and C_g are given by the following relations

$$C_t = \beta_t \sum_{i=1}^{m+1} (W_s)_i \quad (3.3.2)$$

$$C_g = \beta_g \sum_{i=1}^m (W_g)_i \quad (3.3.3)$$

where,

β_t is the cost of tower steel per kN;

β_g is the cost of high tensile steel used for guys per kN;

$(W_s)_i$ is the weight of the i th span in kN;

$(W_g)_i$ is the weight of all the guys at i th level in kN;

and m is the number of guy levels.

3.4 CHOICE OF THE DESIGN VARIABLES

Utmost care has to be exercised in deciding whether a particular parameter defining the design be included as a design variable. On one hand, consideration of all possible design variables increases the dimensionality of the problem, leading to extra and unnecessary computational effort, on the other, non-consideration of some parameters as design variables

may create discontinuities in the objective function, thus making it difficult to optimize. This dichotomy has been kept in mind, while selecting the design variables in the present work.

The maximum deflection of the tall guyed tower and the maximum stresses in the guys and tower members are dependent on the relative stiffnesses of the tower shaft and the guys. Since the guys are pretensioned, the stiffness of the guys is a function of the amount of pretension in the guys besides the guy diameters and the slope of the guy chords. The relative stiffnesses of the tower shaft and the guys are also a function of the location of the joint between the tower shaft and the guys.

Thus, from the above discussion, it is clear that the guy slopes, the guy diameters, the initial guy tensions and the height of the guy levels are obvious choice to be design variables.

Considering a design variable corresponding to each member of the tower, results into a large number of design variables. Practical considerations also do not permit providing different sizes for each and every member. Therefore, a possible way of reducing the number of

design variables is to select three design variables per span of the tower. There are three types of members in each span viz. the leg member, the diagonal bracing member and the web bracing member. Areas of cross-section corresponding to each one of these members in a span are chosen as design variables.

The number of guy levels is also one of the important considerations in the design of tall guyed towers. The larger the number of guy levels for a particular tower, the more would be the ground space covered by the tower. From structural point of view, a tower with more number of guy levels will be experiencing more axial thrust and hence, will undergo more destabilising effect. However, increase in number of guy levels increases the flexural stiffness of the guyed tower. Therefore, the number of guy levels is also a potential candidate to be a design variable.

As the number of ^{Guy levels} ~~design variables~~ is an integer quantity, if it is considered as a design variable, the resulting problem will turn out to be a mixed integer programming problem, the solution of which is not easy to obtain. It is, therefore, natural to look into the

ways of avoiding the incorporation of the number of guy levels as explicit design variable. The number of guy levels is directly dependent upon the height of the guyed tower. The height of the tall guyed towers in use varies from 100 m to 400 m. It is observed that, 1) the variation in the number of guy levels for this range of height is 3 to 7 and 2) higher the height of the tower, more the number of guy levels. As such, fixing the number of guy levels in itself becomes a subject matter of study to be approached from the over-all stability and stiffness considerations. However, this has not been studied in the present work and the number of guy levels is fixed depending upon the height of the tower, based on the experience of the earlier investigators.

It is observed that the maximum stresses in the bracing members is almost invariant over the height of the tower. Therefore, it is not necessary to have $2(m+1)$ design variables corresponding to the areas of cross-section of web and diagonal braces in all the tower spans. Instead a total of only two design variables have been introduced viz. area of cross-section of diagonal and web braces. Though the areas of cross-section

of standard steel section vary discretely, it has been assumed that the design variables corresponding to the areas of cross-section of the members are continuous in nature. The properties of the sections corresponding to these values are appropriately interpolated, from the properties of the available standard sections.

Thus the simplified automated optimum design problem for tall guyed towers will have $5m + 3$ or $5m + 2$ design variables depending upon whether the tower has a cantilever at the top or not. The design variables in the present work are listed below:

1. Slopes of guy chords with the horizontal
(ω_i , $i = 1, \dots, m$)
2. Initial guy tensions (t_i , $i = 1, \dots, m$)
3. Guy diameters (d_i , $i = 1, \dots, m$)
4. Heights of the guy levels (h_i , $i = 1, \dots, m$)
5. Areas of cross-section of the leg members
of the tower (A_i , $i = 1, \dots, m$ or $m+1$)
6. Areas of cross-section of the diagonal braces (A_d)
7. Areas of cross-section of the web braces (A_w)

where,

m is the number of guy levels;

and

the subscript i indicates the i th guy level.

In the design vector, the variables are grouped in the order given above. In each variable type, the variables are arranged in the increasing order of guy levels, from the base of the tower.

3.4.1 Non-dimensionalization of Design Variables:

All the design variables are non-dimensionalized such that they are free of units and lie in the range of 0 to 1. Thus,

$$(x_n)_i = \frac{x_i - (x_{\min})_i}{(x_{\max})_i - (x_{\min})_i} \quad (3.4.1)$$

where,

$(x_n)_i$ is the non-dimensionalized design variable;

$(x_{\max})_i$ is the upper limit on the i th design variable;

$(x_{\min})_i$ is the lower limit on the i th design variable;

and x_i is the i th element of the actual design vector.

Such a normalization puts equal weightage on all the design variables during the search for optimum design. The upper and lower bounds on the design variables, used in the present work, are given in Table 3.1.

TABLE 3.1 : LIMITS ON THE DESIGN VARIABLES

Description	No. of Variables	Units	Lower Limit	Upper limit
Guy slope (ω_i)	m	degrees	10.0	60.0
Guy tension (t_i)	m	kN	0.0	Breaking strength of the guy
Guy diameter (d_i)	m	mm	9.53	101.60
Height of the guy level (h_i)	m	meters	0.00	Total height of the tower
Areas of leg members (A_i)	m+1 or (m)	Sq.cm.	5.00	500.00
Areas of web brace (A_w)	1	Sq.cm.	2.95	93.80
Areas of diagonal brace (A_d)	1	Sq.cm.	2.95	93.80

3.5 CONSTRAINTS

A design is considered to be safe only when it performs within certain limits. This imposes certain constraints on the behaviour parameters of the structure,

which in turn are a function of the design variables. Such constraints are termed as behaviour constraints. Over and above the behaviour constraints, there will always be bounds on the design variables arising out of practical and other considerations. These are termed as side constraints. In addition to these, there are some constraint equations on the design which are required to be satisfied as a result of the particular method of analysis used in the present work. The various constraint equations imposed on the minimum cost design of tall guyed towers are grouped herein.

1. For H in Eq. 2.3.2 to be real, the expression under the square root sign has to be positive. It, therefore, follows that

$$4 t_i^2 - W_i^2 \cos^2 \omega_i \geq 0, \quad i = 1, 2, \dots, m \quad (3.5.1)$$

2. a_1 in Eq. 2.3.14a is real and is strictly positive. Hence, the expression under the square root sign shall be positive. Therefore,

$$(a_2^2 + h_t^2 - h_1^2) > 0, \quad i = 1, 2, \dots, m \quad (3.5.2)$$

where,

all these quantities are described in section 2.3.

3. If the guys are to be safe against tension, the maximum tension in the guys shall follow:

$$\gamma_f (t_{\max})_i \leq \frac{(t_{\text{br}})_i}{\gamma_m}, \quad i = 1, 2, \dots, m \quad (3.5.3)$$

where,

γ_f is the load factor;

γ_m is the material reduction factor; and

$(t_{\max})_i$ is the maximum tension in the guys at i th level

$(t_{\text{br}})_i$ is the breaking tension for guys at i th level.

4. In order that the members of the tower are safe against yielding, the actual maximum stress in any of the members shall not exceed the maximum allowable stress in that member. Hence,

$$(\sigma'_1)_i \leq (\sigma_1)_i, \quad i = 1, 2, \dots, m+1 \quad (3.5.4a)$$

$$\sigma'_d \leq \sigma_d \quad (3.5.4b)$$

$$\sigma'_w \leq \sigma_w \quad (3.5.4c)$$

where

$(\sigma'_1)_i$ is the actual maximum stress in the leg member in the i th span;

$(\sigma_1)_i$ is the maximum allowable stress for any of the leg members in the i th span;

σ_d is the actual maximum stress among all the diagonal bracing members of the tower;

σ_d is the allowable stress in the diagonal bracing members of the tower;

σ_w is the actual maximum stress among all the web bracing members of the tower;

and σ_w is the allowable stress in the web bracings of tower.

5. As per the Indian Standard Specifications [12], the slenderness ratio of the members of the tower has to be limited to 250. This ensures safety of the individual members against buckling. It follows that

$$(\sigma_1)_i \leq 250 \quad i = 1, 2, \dots, m+1 \quad (3.5.5a)$$

$$s_d \leq 250 \quad (3.5.5b)$$

$$\text{and } s_w \leq 250 \quad (3.5.5c)$$

where, $(\sigma_1)_i$ is the maximum slenderness ratio of the leg members in the i th span;

s_d is the maximum slenderness ratio of the diagonal bracing members;

and s_w is the maximum slenderness ratio of the web bracing members.

6. For satisfactory performance of the tower, the deflection of the tower has to be limited. As per present practice, the deflection of the tower, shall not exceed $1/115$ th of the total height of the tower. So, the deflection constraint is formulated as:

$$\delta_{\max} \leq \frac{H_t}{115} \quad (3.5.6)$$

where,

δ_{\max} is the maximum deflection of the tower;

and

H_t is the total height of the tower.

7. In order to ensure that the spans take a non-negative value, we have

$$h_i \geq 0 \quad (3.5.7)$$

$$h_i - h_{i-1} \geq 0, \quad i = 2, 3, \dots, m$$

where,

h_i is the elevation of the i th guy level.

8. Bounds on the design variables: All the design variables are considered to be continuously varying between the lower and upper bounds given by:

$$x_i \geq (x_{\min})_i, \quad i = 1, 2, \dots, N_d \quad (3.5.8)$$

$$x_i \leq (x_{\max})_i, \quad i = 1, 2, \dots, N_d$$

where, N_d is the total number of design variables.

3.5.1 Normalization of Constraints

The constraint equations are normalized for the same purpose as the design variables. Typical examples of normalizing a behaviour and side constraint are shown below:

A behaviour constraint is typically represented as:

$$\delta \leq \delta_{\max}$$

Rewriting the above equation

$$\delta - \delta_{\max} \leq 0$$

Dividing both sides by δ_{\max} , the normalized constraint can be obtained as

$$g_j = \delta / \delta_{\max} - 1 \leq 0 \quad (3.5.9)$$

A general side constraint can be written as:

$$Y_{\min} \leq Y \leq Y_{\max}. \quad (3.5.10)$$

Taking the first part of the inequality and dividing by $(Y_{\max} - Y_{\min})$ we have

$$\frac{Y - Y_{\min}}{Y_{\max} - Y_{\min}} \geq 0$$

$$\text{or} \quad Y_n \geq 0 \quad (3.5.11)$$

$$\text{where } Y_n = \frac{Y - Y_{\min}}{Y_{\max} - Y_{\min}}$$

treating the other part of the inequality in a similar way we get

$$Y_n - 1 \leq 0 \quad (3.5.12)$$

Eqs. 3.5.11 and 3.5.12 can be combined to give

$$g_j = Y_n (Y_{n-1}) \leq 0 \quad (3.5.13)$$

This inequality, Eq. 3.5.13, satisfies both the inequalities in Eqs. 3.5.11 and 3.5.12 simultaneously. Thus, the two constraints on the bounds of a design variable are reduced to only one.

3.6 MINIMIZATION ALGORITHM

The mathematical programming problem formulated in the previous sections is a non-linear programming problem. Though there are many direct and indirect methods available for solving a wide variety of non-linear programming problems, an Interior Penalty Function method is chosen for the present work. This is a very powerful method and it has the advantage of giving all points in the feasible or acceptable design space, and thus advantageous from engineering point of view. The method used in the present work is briefly described in the following section.

3.6.1 Sequential Unconstrained Minimization Technique:

The technic of optimal seeking, developed by Fiacco and Mc Cormick [7], known as the Sequential Unconstrained Minimization Technique (SUMT) is an Interior Penalty Function method. In this method, the constrained problem is transformed into an equivalent unconstrained problem, by appending the constraints to the objective function through a penalty parameter, r . This gives a sort of imaginary wall at the constraint surface which does not permit crossing over of the design into the infeasible domain.

Thus, the constrained problem in Eq. 3.1.1 is transformed to

$$\text{Minimize } F(\vec{X}, r) = \overbrace{f(\vec{X})}^{\text{}} + r \sum_{j=1}^m \frac{1}{g_j(\vec{X})} \quad (3.6.1)$$

in which,

$F(\vec{X}, r)$ is known as Penalty Function.

This function $F(\vec{X}, r)$ is minimized for decreasing sequence of the penalty parameter, r . The sequence of minima so obtained converge to the minimum of the constrained problem as $r \rightarrow 0$.

The initial value of the penalty, parameter, r_0 , has to be carefully chosen for better convergence. It is customary to choose r_0 such that the objective function and the penalty are of the same order of magnitude at the starting point. Thus

$$r_0 = \left| \frac{f(\vec{X})}{-\sum 1/g_j} \right| \quad (3.6.2)$$

Subsequently, the penalty parameter can be reduced by a constant reduction factor.

The unconstrained minimization of the penalty function, $F(\vec{X}, r)$, is performed by Powell's method [19]. The objective function expression is a continuous function of design variables and the penalty function has been found to exhibit discontinuities. This necessitated the use of a non-gradient method for unconstrained minimization. Powell's method, being the most powerful among the non-gradient methods, is chosen for the unconstrained minimization. In this method, each iteration requires n one-dimensional (linear) minimizations in n linearly independent directions $\bar{S}_1, \bar{S}_2, \dots, \bar{S}_n$. As the result of these minimizations a new direction, \bar{S} is defined and, if a test is passed, \bar{S} replaces one of the original directions, \bar{S}_i . The initial \bar{S}_i 's are taken as coordinate directions.

The procedure for minimizing an unconstrained function $f(\vec{X})$ from a starting point \vec{X}_0 , is as follows:

1. For $i = 1, 2, \dots, m$ calculate α_i^* so that $f(\vec{X}_{i-1} + \alpha_i^* \vec{S}_i)$ is a minimum and define $\vec{X}_i = \vec{X}_{i-1} + \alpha_i^* \vec{S}_i$.
2. Find the integer $k, 1 \leq k \leq n$, so that $|f(\vec{X}_{k-1}) - f(\vec{X}_k)|$ is a maximum, and define $\Delta = f(\vec{X}_{k-1}) - f(\vec{X}_k)$.
3. Calculate $f_3 = f(2\vec{X}_n - \vec{X}_0)$ and define $f_1 = f(\vec{X}_0)$ and $f_2 = f(\vec{X}_n)$.
4. If either $f_3 \geq f_1$ and /or

$$(f_1 - 2f_2 + f_3) \cdot (f_1 - f_2 - \Delta)^2 \leq 1/2 \Delta (f_1 - f_3)^2$$

Use the old directions $\vec{S}_1, \vec{S}_2, \dots, \vec{S}_n$ for the next iteration and use \vec{X}_n for the next \vec{X}_0 .

5. Otherwise, defining $\vec{S} = \vec{X}_n - \vec{X}_0$, calculate α^* so that $f(\vec{X}_n + \alpha^* \vec{S})$ is a minimum. Use $\vec{S}_1, \vec{S}_2, \dots, \dots, \vec{S}_{k-1}, \vec{S}_{k+1}, \vec{S}_{k+2}, \dots, \vec{S}_n, \vec{S}$ as the directions for the next iteration and $\vec{X}_n + \alpha^* \vec{S}$ for the next \vec{X}_0 .

The linear minimization to find α_i^* is performed by the Golden Section Search. In this method, the interval of uncertainty, within which the minimum is bracketed, is reduced by 0.618 at every iteration, by deleting the region in which the minimum is not expected to occur. This is the most efficient linear search technique which makes use of only function values.

The algorithms of these methods are described in detail in any standard text book on non-linear programming [9], [20].

CHAPTER IV

RESULTS DISCUSSIONS AND CONCLUSIONS

General computer programs have been developed for the analysis and optimum design of multi-level guyed towers. The results obtained are presented in this chapter. All the results presented herein are obtained on DEC-1090 system at the Computer Centre, I.I.T. Kanpur.

4.2 RESULTS

Optimum design of a 3 level 100 m high triangular tower with a cantilever at the top has been carried out in the present work. The optimum design problem of this structure turns out to be a non-linear programming problem with 18 design variables and 42 constraints. The components of the design vector and the order of the constraints is given in Table 4.1 and 4.2 respectively.

The base of the tower is assumed to be hinged and no antenna loads are considered on the tower. Other salient data considered in the present work is:

Basic Wind Velocity (V_0) = 160 kmph

Wind Velocity Exponent (α) = 1/8

Total height of the tower (H_t) = 100.0 m

Tower face width (S)	= 5.0 m
Panal heights of the tower	= 2.5 m
Number of guy levels (m)	= 3
Number of guys per panal	= 3
Cost of high tensile steel ropes (β_g)	= Rs. 1400/kN
Cost of High Yield Strength Deformed (HYSD) Steel (β_t)	= Rs. 400/kN
Modulus of elasticity of tower steel	= 2.1×10^5 MPa
Modulus of elasticity of guy rope	= 1.38×10^5 MPa
Load factor (γ_f)	= 1.20
Material reduction factor (γ_m)	= 1.15
Convergence limit on the design variables	= 0.001
Convergence limit on the function values	= 0.001

Same values of convergence limit are used in all the optimization routines.

Minimum cost design of the tower has been worked out from two different starting points. These starting points were obtained by trial and error. One of these starting points, henceforth addressed as SP-1, is a free point while the other, addressed as SP-2, is a bound point. The design vectors at the end of each unconstrained minimization of the penalty function, starting from SP-1 and SP-2 are given in Tables 4.3

and 4.4 respectively. Other salient data is given in Tables 4.3a and 4.4a. The starting value of penalty parameter, r , is determined from Eq. 3.6.2 and the reduction in the penalty parameter is taken as 0.1, from one unconstrained minimization to the other.

4.3 DISCUSSIONS

The starting design point in Table 4.3, SP-1, is a free point. The first cycle of minimization results into the design given in column 3. Column 7 indicates the proposed minimum cost design of the guyed tower considered. Comparing the designs in columns 3 and 7, it is observed that only the design variables 7, 8, 13, 14, 15 and 16 have changed considerably during the last 4 cycles of sequential unconstrained minimization. Starting from SP-1, the first cycle of optimization process has brought the design variables corresponding to the guy slopes, the guy tensions, the location of guy levels and the areas of cross-section of the bracing members to the values, which more or less, correspond to the minimum cost design. It is only the design variables corresponding to the guy diameters of guys at levels 1 and 2 and the areas of cross-section

of the leg members in all the spans, which vary in the subsequent sequential minimization process, leading to the reduction in the cost of the structure. The variables 10, 11 and 12 corresponding to the location of the guy levels and 17 and 18 corresponding to the areas of cross-section of the diagonal and web bracings have remained more or less constant throughout the optimization process. The value of the variable 16, corresponding to the area of cross-section of the leg members in the cantilever span has slightly increased, from 40.000 to 40.248, in the first cycle of minimization and then decreased continuously. However, the values of other variables, corresponding to the areas of cross-section of the leg members in the other spans, have decreased even in the first cycle. From column 7 of Table 4.3, it can be observed that corresponding to the minimum cost design: 1) the slope of the guy chords increase for higher levels, 2) the initial guy tensions at all levels are more or less of the same magnitude, 3) the areas of cross-section of the leg members of the tower decrease from the base of the tower to the tip of it.

Table 4.3a indicates the values of the penalty parameter, penalty function, objective function and the cost of the tower shaft and guys for intermediate designs; starting from the initial design SP-1. The active constraints for these designs are also indicated in this table. It is observed that, the objective function and the cost of the tower shaft have decreased monotonically. The cost of the guys has increased in the 1st cycle of minimization and then decreased monotonically. This increase is due to the increase in the value of the variable 8, corresponding to the guy diameter at the 2nd level. For the final design the constraints 36, 37, 38 and 39 corresponding to the stresses in the leg members of the tower are active. In other words, stress limitation in the leg members governs the minimum cost design of the guyed tower considered. The total CPU time taken to reach the proposed optimum design is around one hour fifteen minutes.

The starting design point Table 4.4, SP-2, is a bound point. The maximum deflection at the tip of the tower is bounded at this point. Comparing the starting design point, column 2, and the proposed minimum

cost design, column 7 of Table 4.4, it is seen that, there is a considerable change in the values of all the design variables, except for that of variables 10, 11 and 12, corresponding to the location of the guy levels, and the variables 17 and 18, which correspond to the areas of cross section of the bracing members. The guy slopes, which have initially been in the decreasing order of magnitude from the base of the tower to the top of it, have reversed in order in the final design. The initial guy tensions have increased from a lower value in the beginning to a higher value at the end. The diameters of the guys at all levels have more or less decreased.

Areas of cross-section of the leg members in all the other spans have decreased continuously, where as, the area of cross-section of leg members in the cantilever span has increased, from 19.635 to 44.106, in the first cycle of minimization and then decreased continuously.

This is to move the design away from the composite constraint surface in the initial stages of optimization. The variables corresponding to the location of the guy levels have remained unchanged in the entire process of optimization. The variables 17 to 18, viz. The areas of cross-section of the bracing members, remained more or less unchanged.

The variables corresponding to the guy slopes, the initial guy tensions and the areas of cross-section of the leg members have experienced a considerable change in their values in the initial stages of the optimization and more or less remained unchanged subsequently. However, the change in the values of the guy diameters is brought about in the final stages of the process. The total CPU time taken to reach the proposed optimum design is around one hour.

Table 4.4a indicates the values of the penalty parameter, penalty function, objective function and the cost of the tower shaft and guys for the designs at the end of each unconstrained minimization starting from the design SP-2. This table also indicates the active constraints at these designs. A monotonic decrease in the value of the objective function and the cost of the tower shaft and guys is indicated in the table. For the initial design, the constraint 42, corresponding to the deflection of the tower, is active. It can be observed that for the intermediate designs at the end of cycles 1, 2 and 3 none of the constraints are active and for the final design, the constraints 26, 35, 36, 37 and 38 corresponding to the L/R ratio of the web bracing, stress in the guys at the third level and stresses in the leg

members in spans 1, 2 and 3 respectively, are active. Thus the optimization process has initially moved the design away from the constraint, corresponding to the limitation on the deflection, bringing the design closer to the stress constraints subsequently. The proposed minimum cost design obtained from the starting point SP-2 is also a stress constrained design, as indicated by Table 4.4a. However, this design is very different, both in details as well as in the overall cost, as compared to the proposed minimum cost design obtained by starting from SP-1.

Table 4.5 indicates the number of linear minimizations required for convergence by each of the unconstrained minimization cycles, from both SP-1 and SP-2 as starting designs. It is observed that, the number of unconstrained minimizations required increase upto the third cycle and from the fourth cycle onwards, this number decreases. This is because, by this time, the design has reached very near to the optimum, where, the function is better approximated by a quadratic. The fifth cycle of unconstrained minimization is performed only to ensure the convergence of the process in both the cases. The process with SP-1 as starting point, requires more effort than with SP-2, as is clear from Table 4.5.

As the locations of the guy levels have not changed during the process of optimization, it is not necessary to consider the locations of the guy levels as design variables, while seeking the optimum solutions of guyed towers. However, different sets of the locations of the guy levels have given different optimal solutions, as is evident from Table 4.3 and 4.4. Moreover, the location of the guy levels has a considerable effect on the stability and flexural stiffness of the tower. Therefore, a separate study has to be made to fix the locations of the guy levels viz. the heights of the guy levels, before embarking on the optimization process. In otherwords, an optimal configuration of the guyed tower has to be evolved before detailing of the members is done to obtain minimum cost design.

Both the starting designs, SP-1 and SP-2, are very different and the solutions obtained from these are also different. The optimum obtained from SP-1 is costlier than that from SP-2. This indicates the existence of local minima. For proposing a global minimum with some degree of confidence, optimum solutions have to be obtained from a number of different starting designs. This, however, has not been possible in the present work

due to prohibitive cost of computer time required in obtaining an optimum solution. The most of the computer time is taken up by the non-linear structural analysis of the guyed tower.

From the numerical results obtained, the following observations are made for the proposed optimum design:

1. The slopes of the guy chords increase as we move above the base of the tower.
2. The areas of cross-section of the leg members decrease from the base to the tip of the tower. This is justifiable because, the loads on the tower shaft increase from top to bottom.
3. The initial guy tensions are nearly of the same magnitude at all levels.
4. During the process of optimization, the change in the variables corresponding to the location of the guy levels is insignificant.
5. The cost of the guys is about 20 per cent of the cost of the tower shaft at the proposed minimum cost design in both the cases.

4.4 CONCLUSIONS

An automated optimum design of tall multi-level guyed towers has been formulated and some numerical results have been illustrated in the present work. From the present experience, the following conclusions can be drawn.

1. The optimum design is quite sensitive to the number and location of the guy levels, as this changes the magnitude of loads in various parts of the structure. Therefore, configuration optimization has to precede the detailing of the design variables at the optimum.
2. The slopes of the guy chords flatten towards the base of the tower, at the optimum.
3. The areas of cross-section of the leg members for an automated optimum design of the tower, decrease towards the tip of the tower.
4. The problem of automated optimum design for tall guyed towers exhibits local minima.
5. Maximum deflection of the guyed tower does not govern the minimum cost design of tall guyed towers. On the contrary, the minimum cost design turns out to be stress constrained.

4.5 SCOPE FOR FURTHER WORK

1. The performance of the tower is quite sensitive to the number and location of the guy levels, because the flexural stiffness and the over-all stability of the structure are effected by the number and location of guy levels. Therefore, a study to fix the number and location of the guy levels, preceding the detailed optimum design, has to be carried out.
2. In the present work, only wind loading on the tower is considered. Antenna loading should be incorporated inorder to simulate realistic loading conditions.
3. In the present work, it has been assumed that the bracing members, in all the spans, have equal areas of cross-section. Consideration is to be given to vary the area of cross-section of bracing members in each span.
4. The panel heights of the tower in each span have also to be taken as design variables and its effect has to be studied on the optimum design.
5. Cost of the ground space covered by the tower and cost of the foundations are not considered herein.

The inclusion of these may change the over-all design of the tower. Therefore, these aspects also have to be incorporated in the automated optimum design framework.

6. Last but not the least is to develop a reasonably accurate and computationally efficient method of reanalysis of the guyed tower, which, has inhibited the incorporation of most of the points mentioned above.

TABLE 4.1 : COMPONENTS OF THE DESIGN VECTOR

Variable number(s)	Description
1 to 3	Guy slopes
4 to 6	Initial guy tensions
7 to 9	Guy diameters
10 to 12	Guy level heights
13 to 16	Areas of leg members
17	Areas of diagonal braces
18	Areas of web braces

TABLE 4.2 : COMPONENTS OF THE VECTOR OF CONSTRAINTS

Constraint number(s)	Description
1 to 18	Bounds on the design variables
19 to 20	Restrictions on the tower spans 2 and 3 respectively
21 to 24	Limits on maximum L/R ratios of leg members in spans 1,2,3 and cantilever respectively
25	Limit on maximum L/R ratio of diagonal bracing
26	Limit on max. L/R ratio of web bracing.
27 to 29	Eq. 3.5.1
30 to 32	Eq. 3.5.2
33 to 35	Constraints on the max. stress in the guys at level 1, 2 and 3 respectively.
36 to 39	Constraints on the max. stress in the leg members in spans 1,2,3 and the cantilever respectively.
40	Constraint on the max. stress in the diagonal bracings.
41	Constraint on the max. stress in the web bracings.
42	Limit on the maximum deflection of the tower

TABLE 4.3 : DESIGNS STARTING FROM SP-1

Variable	Initial value	Value of the variable at the end of Iteration				
		1	2	3	4	5
X(1)	30.000	30.060	30.170	30.170	30.145	30.145
X(2)	42.500	42.454	42.971	43.334	43.334	43.334
X(3)	55.000	52.315	52.240	52.215	52.500	52.500
X(4)	100.000	119.585	119.193	119.193	119.193	119.193
X(5)	100.000	119.302	119.302	117.767	117.767	117.767
X(6)	100.000	101.021	101.021	101.652	104.392	104.392
X(7)	25.000	24.915	21.412	19.931	19.735	19.735
X(8)	25.000	29.488	24.188	24.032	24.032	24.032
X(9)	25.000	25.000	24.876	24.655	24.655	24.655
X(10)	20.000	19.950	19.900	19.900	19.900	19.900
X(11)	50.000	49.950	49.900	49.900	49.900	49.900
X(12)	85.000	84.950	84.950	84.950	84.950	84.950
X(13)	100.000	75.038	57.904	53.114	52.299	52.299
X(14)	90.000	70.084	55.026	50.608	49.678	49.678
X(15)	75.000	67.145	52.385	47.058	45.632	45.632
X(16)	40.000	40.248	27.816	25.914	25.496	25.496
X(17)	10.470	10.425	10.340	10.295	10.295	10.295
X(18)	8.000	8.045	8.136	8.202	8.202	8.202

TABLE 4.3.a : DATA OF INTERMEDIATE DESIGNS STARTING FROM SP-1

Design at the end of iteration number	Penalty parameter	Value of the Penalty function at the start of the iteration	Values at the end of the iteration				
			Penalty function	Objective function	Cost of tower shaft	Cost of guys	Numbers of active constraints
0*	612.11	-	301570	150785	129810	20975	-
1	612.11	301570	287723	139890	115531	24359	-
2	61.211	154674	138750	119857	99954	19902	-
3	6.1211	121667	114555	117230	97935	19295	-
4	0.61211	114832	114001	113430	94257	19173	36, 37, 38 and 39
5	0.061211	113472	113472	113430	94257	19173	36, 37, 38 and 39.

* Iteration 0 corresponds to the starting design.

TABLE 4.4 : DESIGNS STARTING FROM SP-2

Variable	Initial Value	Value of the variable at the end of iteration				
		1	2	3	4	5
X(1)	37.500	37.450	37.588	37.667	37.667	37.667
X(2)	32.500	32.500	40.667	40.600	40.600	40.600
X(3)	25.000	34.384	45.156	45.256	45.256	45.256
X(4)	39.960	82.346	80.935	80.545	79.673	79.673
X(5)	53.300	87.262	86.872	86.092	86.482	86.482
X(6)	39.950	89.198	88.808	87.546	86.525	86.525
X(7)	20.640	20.640	20.594	19.229	18.878	18.878
X(8)	20.640	20.760	20.715	20.064	19.856	19.856
X(9)	20.640	20.136	19.358	18.719	18.460	18.460
X(10)	25.000	25.000	25.000	25.000	25.000	25.000
X(11)	60.000	60.000	60.000	60.000	60.000	60.000
X(12)	90.000	90.000	90.000	90.000	90.000	90.000
X(13)	122.718	46.647	43.751	42.896	42.572	42.572
X(14)	95.033	43.031	43.031	40.930	40.377	40.377
X(15)	50.266	36.592	33.376	32.130	31.712	31.712
X(16)	19.635	44.106	20.325	20.325	19.907	19.907
X(17)	7.270	7.270	7.270	7.242	7.242	7.242
X(18)	5.270	5.270	5.270	5.012	4.921	4.921

TABLE 4.4a : DATA OF INTERMEDIATE DESIGNS STARTING FROM SP-2

Iteration number	Penalty parameter	Penalty function at the start of the iteration	Values at the end of the iteration				Numbers of active constraints
			Penalty function	Objective function	Cost of tower shaft	Cost of guys	
0*	259.87	-	282346	141173	116527	24646	42
1	259.87	282346	191211	96710	76113	20597	-
2	25.987	106171	99833	88512	71929	16583	-
3	2.5987	89643	86941	85262	69898	15364	-
4	0.25987	85422	84594	84200	69243	14957	26, 35, 36, 37 and 38
5	0.025987	84224	84224	84200	69243	14957	26, 35, 36, 37 and 38

* Iteration number 0 corresponds to the starting design.

TABLE 4.5 : NUMBER OF LINEAR MINIMIZATIONS PER CYCLE

Order of unconstrained minimization cycle	Number of linear minimizations required for convergence	
	Starting from SP-1	Starting from SP-2
1	54	54
2	90	72
3	108	90
4	36	36
5	18	18

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APPENDIX - I A

 DESCRIPTION OF COMPUTER PROGRAMS

The computer programs developed in the present work consist of a main program, which initiates the program execution, and 17 other subroutines which are explained below.

- INPUT :** This subroutine reads the tower data, calculates the number of variables and number of constraints and establishes the design vector. It gets the design vector normalised through the subroutine NORMAL. This program is called only once during the execution, the calling program being the main program.
- PENALT :** This segment performs constrained minimization using interior penalty function method.
- POWELL :** This program is called by PENALT and performs unconstrained minimization of the penalty function using Powell's method.
- GOLD :** This module is called by POWELL for performing linear minimization. It uses the Golden Section Search technique for one-dimensional minimization.
- OUT :** OUT is a program for outputting the design vector and the values of the penalty function and objective functions at various stages of unconstrained minimization.
- OUTPUT :** This program outputs the tower particulars at various stages in the process of optimization.
- FUN :** Program FUN does the following operations :
 1) Establishes and checks the constraints, 2) calls the design programs (MEMBER, LEG, GUYDIM) for calculating the weights and strengths of various components of the guyed tower, 3) if the side-constraints are satisfied calls the analysis program (POWER) 4) establishes and checks the behaviour constraints and computes the objective function and the penalty function.
- MEMBER :** The properties of standard rolled steel equal angle sections are stored by this program. Given the area of cross section and length of the member, this program returns the strength and l/r ratio of the section.
- LEG :** In this program the properties of HYSD round bars are stored. This program also outputs the strength and l/r ratio of a given section.

10. GUYDIM : The properties of wire ropes are contained in this program. It returns the weight and strength of a given section of guy.
11. NORMAL : This module normalizes a given design vector, using the upper and lower bounds on the design variables.
12. ACTUAL : This program converts a normalized design vector to its corresponding actual vector.
13. TOWER : TOWER is called by the subroutine FUN. It establishes the loads on the tower-shaft and the guys, performs the iterative cycle of analysis, calculates the stresses in the tower members, slack stresses in the guys and deflections of the tower. The maxima of these values are returned to the calling program.
14. VEL : VEL calculates the wind velocities on the guys.
15. WLOAD : The wind loads on the tower-shaft are calculated in this segment.
16. CONST : This program calculates the spring constants of the guys in the wind ward and normal to wind directions.
17. SETS : The simultaneous linear equations that are encountered in the analysis are established and solved by this program.

APPENDIX - C -----

DETAILS OF INPUT DATA -----

DESCRIPTION	UNITS	VARIABLE
1. TITLE CARD		
Title		PJW
2. GUY LEVEL CARD		
Number of Guy Levels		NG
Base Fixity		IBASE
Hinged = 01		
Fixed = -1		
Stress Computation	-	NT
Always = -1		
Wind Direction	-	IA
Face Wind = 01		
Corner Wind = -1		
Side Wind = 02		
Modulus of Elasticity	ton/sqcm	R
Problem No. or Case No.	-	CASE
3. WIND DATA CARD		
Wind Velocity	knph	VO
Win Reference Height	mt	HIO
Escalation Cut-Off Height	mt	HIE
Escalation Exponent	-	ALPH
Tower Base Height	mt	HIF
Settlement	mt	HTFA

TABLE 1 (Continued)

DESCRIPTION	UNITS	VARIABLE
4. GUY CARD - ONE PER GUY LEVEL		
Local Vertical Load	tons	WI
Local Horizontal Load Parallel to wind	tons	WN
Number of Guys	-	NN
Groups of Identical Guys	-	NI
Local Moment Parallel to Wind	ton-mt	EXMW
Local Moment Normal to Wind	ton-mt	EXMN
5. ANGLE CARD - SAME AS NUMBER OF GUYS PER LEVEL		
Angle (Counterclockwise)	deg	ANG
6. GUY DIMENSION CARD - ONE PER GROUP OF IDENTICAL GUYS PER LEVEL		
Horizontal Distance (Anchor to Tower)	mt	A1
Vertical Distance (Anchor to Tower)	mt	H
Initial Gut Tension at Anchor	tons	T
Guy Weight without Ice	kg/mt	W
Guy Weight with Ice	kg/mt	WR
Guy Diameter	cm	D

TABLE 1 (Continued)

DESCRIPTION	UNITS	VARIABLE
Product of Area and Modulus of Elasticity of Guy	tons	G
Lever Arm	mt	X3
No. of Insulators	-	NIN
Length of One Insulator	mt	ELIN
Weight of One Insulator	tons	WIN
No. of Identical Guys in Group	-	N
7. TOWER CARD - ONE PER SPAN		
Length of Span	mt	EL
Average Moment of Inertia of the Span	$\frac{2}{2}$ cm-mt	AI
Weight of Span	tons	WS
Drag Area	sqmt /mt	C
8. PULL-OFF CARD		
Horizontal Pull-Off Load Parallel to Wind	tons	TH
Pull-Off Moment parallel to Wind	tons-mt	TM
Pull-Off Vertical Load Component	tons	T
Horizontal Pull-off Load Normal to Wind	tons	TH1
Pull-Off Moment Normal to Wind	tons-mt	TM1

APPENDIX 1 (CONTINUED)
 =====

DESCRIPTION	UNITS	VARIABLE
9. TEMPERATURE CARD		
Load Case Temperature	Degrees F	B
Normal Temperature	Degrees F	C
10. WIDTH CARD		
Tower Face Width	mt	S
11. PANEL LENGTH CARD - ONE PER SPAN		
Top Panel Length	mt	YF
Bottom Panel Length	mt	YB
Intermediate Uniform Panel Length	mt	W
Number of Uniform Panels	-	N

APPENDIX - D
 PROPERTIES OF WIRE ROPES

Rope Diameter in mm	Minimum breaking Strength in kN	Net Metallic Area in sq.mm	Weight of Rope in N/m
9.53	57.8	41.9	3.53
11.11	78.4	58.7	4.70
12.70	102.0	76.8	6.08
14.29	129.4	94.8	7.74
15.88	159.8	117.4	9.51
17.46	191.2	142.6	11.56
19.05	231.4	172.4	13.82
20.64	266.7	200.7	16.07
22.23	311.8	232.9	18.62
23.81	355.9	267.1	21.46
25.40	406.9	303.9	23.52
28.58	513.8	384.5	30.77
31.75	642.3	480.7	38.51
34.93	781.5	584.6	46.84
38.10	924.7	694.2	55.66
41.28	1098.0	819.4	65.76
44.45	1275.0	948.4	76.44
47.63	1461.0	1090.4	87.91
50.70	1657.0	1238.8	99.86
53.98	1863.0	1400.1	112.70
57.15	2089.0	1561.4	126.32
60.33	2324.0	1735.6	140.14
63.50	2559.0	1916.2	154.55
66.68	2824.0	2109.8	169.44
69.85	3089.0	2309.8	185.81
73.03	3373.0	2522.7	202.66
76.20	3667.0	2742.1	220.30
82.55	4226.0	3251.8	262.44
88.90	5697.0	3761.5	306.25
95.25	6491.0	4897.1	393.76

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